Intuitive Aspects of the Semantic Computations in a KBO

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Abstract. The concept of *Knowledge Bases with Output* (KBO) was introduced in [2]. In this paper we present several intuitive aspects concerning the semantic computations realized in a KBO. We explain the manner in which the information contained in a knowledge piece is transposed in a labelled graph and then we show how we can build a labelled stratified graph such that a reasoning process can be formalized. If we append an output space and a morphism of partial algebras then we obtain a knowledge base with output. The intuitive aspects with respect to this morphism are presented and all the concepts treated in [1] and [2] are relieved on an example.

1 Introduction

The concept of *labelled stratified graph* was suggested by the graph-based methods for knowledge representation. By generalizing the knowledge representation in a semantic network and using several concepts and results of universal algebra we obtain the concept of *labelled stratified graph*, which is the main component of a KBO. This concept is treated in [1]. If we try to give a simple description of this concept, we say that a KBO is represented by the equation $KBO = LSG + OS$. This means that a KBO is a structure consisting in a labelled stratified graph (LSG) and an *output space* (OS). The elements of OS may be sentences in a natural language, graphical images and so on. In what follows we shall present by means of an example several aspects of the computations in such a structure.

2 Initial knowledge and labelled graphs

By a *knowledge piece* we understand the description of some *world* of objects. The description is given in a natural language. The information specified in a knowledge piece consists of several objects and the relations between them. We shall suppose these relations are binary ones, that is, they are subsets of some Cartesian product. The whole information given in a knowledge piece is named *initial knowledge*.

In this section we explain the manner in which the initial knowledge, can be transposed in a labelled graph. Intuitively, a labelled graph is an oriented graph such that each arc is assigned to some element of a set; this element is called *label*. In a labelled graph several binary relations are obtained. A binary relation is defined by the set of all the pairs of objects such that they are connected by an arc containing the same label. But the cardinal number of the binary relations may be less than the cardinal number of the labels. This can be explained by the fact that may exist several labels for the same binary relation. Such a situation is encountered in the case when a given relation has several meanings in the knowledge piece.

We consider the following knowledge piece KP:

> *Emily is Helen's child. Helen is Ann's sister. Ann is Peter's sister. Emily likes to play tennis with Helen. Ann is the tennis trainer of Emily.*

We observe that this piece relieves several objects: Emily, Helen, Ann and Peter. We denote by $S = \{Emily, Helen, Ann, Peter\}$ the set of the corresponding objects. There are several binary relations between some of them. Obviously we obtain the following binary relations on S:

$\rho_1 = \{(Emily, Helen)\}$,
$\rho_2 = \{(Helen, Ann), (Ann, Peter)\}$,
$\rho_3 = \{(Ann, Emily)\}$

We can transpose KP into some labelled graph $G = (S, L_0, T_0, f_0)$, where

- $S = \{Emily, Helen, Ann, Peter\}$
- $T_0 = \{\rho_1, \rho_2, \rho_3\}$
- L_0 is a set of elements called *labels* for the relations from T_0
- $f_0 : L_0 \longrightarrow T_0$ is a surjective mapping.

The first question that can arise is the following: how many labels contains the set L_0 ? In order to answer this question we analyse every pair of objects and for every distinct meaning of the relation specified in KP we assign a symbol in L_0. Thus, for the pair $(Emily, Helen)$ from ρ_1 we find two meanings: we assign the symbol a_1 for the property "child of" and the symbol b_1 for the property "likes to play tennis". We denote by a_2 the symbol assigned to ρ_2, that is the property "is sister of", and by a_3 the symbol assigned to ρ_3. We obtain the labelled graph drawn in figure 1.

As a consequence we obtain the mapping f_0. Because this mapping shows the assignment described above, we obtain:

$$f_0(a_1) = f_0(b_1) = \rho_1, f_0(a_2) = \rho_2, f_0(a_3) = \rho_3$$

We consider a labelled stratified graph $\mathcal{G} = (G, L, T, \sigma_T, f)$ over G ([1]). We remember that

- $L \subseteq H$, where $H = \bigcup_{n \geq 0} H_n$, $H_0 = L_0$ and $H_{n+1} = H_n \cup \{\sigma(u, v) \mid u, v \in H_n\}$

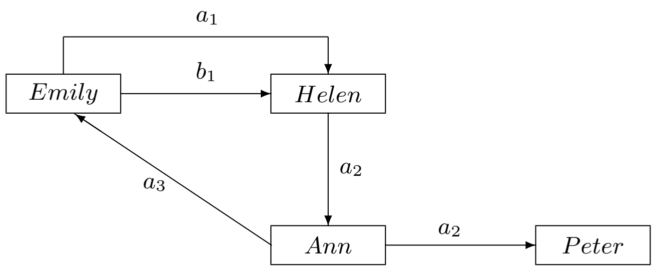
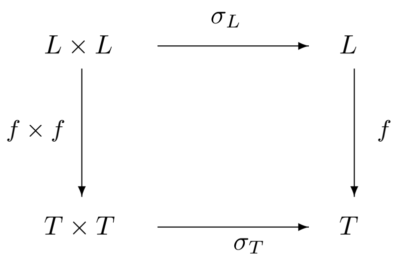


Fig. 1. Labelled graph for KP

- σ_T is a restriction of the mapping $prod_S$, where $prod_S$ is the product operation of binary relations on S
- T is the closure of T_0 with respect to σ_T
- $f : L \longrightarrow T$ is a surjective extension of f_0 such that the following diagram is commutative:



Let us choose a mapping σ_T, a restriction of the operation $prod_S$. The mapping $prod_S$ is represented in table 1, where

$\rho_4 = \{(Emily, Ann)\}$; $\rho_5 = \{(Helen, Peter)\}$
$\rho_6 = \{(Helen, Emily)\}$; $\rho_7 = \{(Ann, Helen, \}$
....

Taking the closure of T_0 with respect to $prod_S$ we obtain

$$\{\rho_1, \rho_2, \rho_3\} \cup \{\rho_4, \rho_5, \rho_6, \rho_7\} \cup \ldots$$

If we take the restriction σ_T of $prod_S$, given in table 2, then the closure T of T_0 under σ_T is the following:

$$T = T_0 \cup \{\rho_4, \rho_5, \rho_6\}$$

We know ([1])that

$$dom(\sigma_L) = \{(u, v) \in L \times L \mid (f(u), f(v)) \in dom(\sigma_T)\}$$

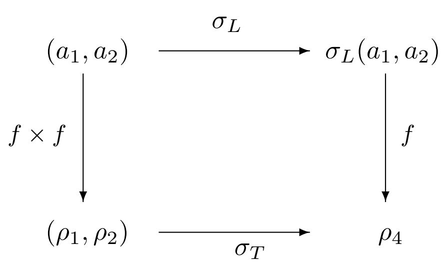
$prod_S$	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	...
ρ_1		ρ_4			ρ_8	
ρ_2		ρ_5	ρ_6			
ρ_3	ρ_7					
ρ_4		ρ_8				
...	...	...				

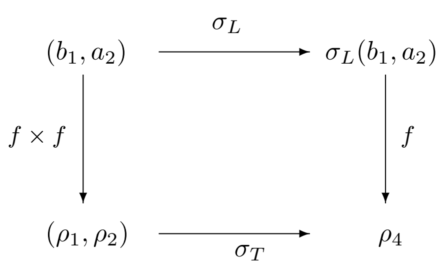
Table 1. The mapping $prod_S$

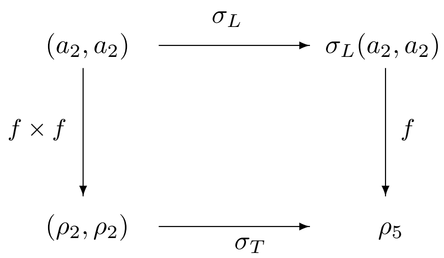
σ_T	ρ_1	ρ_2	ρ_3
ρ_1		ρ_4	
ρ_2		ρ_5	ρ_6
ρ_3			

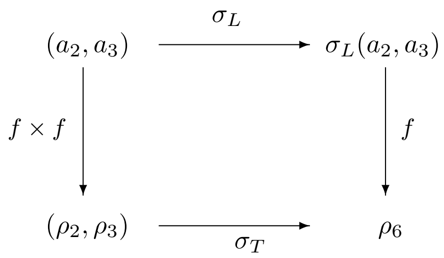
Table 2. The mapping σ_T

The mapping f and the set L can be obtained if we impose the condition of commutativity of the diagram specified in the definition of a labelled stratified graph:









In this way we obtain the set L:

$$L = L_0 \cup \{\sigma(a_1, a_2), \sigma(b_1, a_2), \sigma(a_2, a_2), \sigma(a_2, a_3)\}$$

We shall denote by K_0 the set

$$K_0 = \{(x, a, y) \in S \times L_0 \times S \mid (x, y) \in f_0(a)\}$$

We observe that the elements of K_0 are taken from the labelled graph G: an element (x, a, y) is introduced in K_0 if and only if there is an arc from x to y and the label of this arc is a.

A knowledge base with output (KBO) is a system $KBO = (\mathcal{G}, Y, *, g)$, where ([2])

- $\mathcal{G}$ is a labelled graph
- Y is a set named *output space* and $* : Y \times Y \longrightarrow Y$ is a partial binary operation
- $g : K_0 \longrightarrow Y$

Various elements can be included in the output space Y. In our case we denote by Y a set of sentences of the form:

p_1(x,y)= "x is the child of y"
q_1(x,y)= "x likes to play tennis with y"
p_2(x,y)= "x is the sister of y"
p_3(x,y)= "x is the tennis trainer of y"
p_4(x,y)= "x is the niece of y"

p_5(x,y)= "x is the sister of the tennis trainer of y"

where x and y will be substituted by objects of KP. The partial binary operation $*$ will be defined as follows:

p_1(x,y) $*$ p_2(y,z)= p_4(x,z)
p_2(x,y) $*$ p_2(y,z)= p_2(x,z)
p_2(x,y) $*$ p_3(y,z)= p_5(x,z)
p_4(x,y) $*$ p_2(y,z)= p_4(x,z)

The mapping $g : K_0 \longrightarrow Y$ will be defined by the relations:

$g(x, a_1, y) = p_1(x, y)$
$g(x, a_2, y) = p_2(x, y)$
$g(x, b_1, y) = q_1(x, y)$
$g(x, a_3, y) = p_3(x, y)$

The manner in which the mapping g is defined is obvious: the mapping g specifies the meaning of each symbol from L_0 and this meaning was used when I defined the elements of this set.

Now, we shall extend the mapping g such that for every pair $(x, y) \in S \times S$ and $u \in L$ for which $(x, y) \in f(u)$, $g(x, u, y)$ will represent the element of Y giving the meaning of u. In other words, we extend the mapping g such that g becomes a partial mapping $g : S \times L \times S \longrightarrow Y$: if $g(x, u, y)$ and $g(y, v, z)$ are defined elements in Y and the pair $(g(x, u, y), g(y, v, z))$ belongs to the domains of the partial binary operation $*$ then we take $g(x, \sigma_L(u, v), z) = g(x, u, y) * g(y, v, z)$. We observe that the mapping g "works" as a morphism of partial algebras. In order to relieve this morphism we consider the set

$$\mathcal{T} = \{(x, u, y) \in S \times L \times S \mid (x, y) \in f(u)\}$$

This set becomes a partial algebra if we consider the following binary operation on $\mathcal{T}$:

$$\odot : \mathcal{T} \times \mathcal{T} \longrightarrow \mathcal{T}$$

defined as follows:

1) $\odot$ is defined on the pair $((x, u, y), (y, v, z))$ if and only if $(u, v) \in dom(\sigma_L)$
2) if the previous condition is realized then

$$\odot((x, u, y), (y, v, z)) = (x, \sigma_L(u, v), z)$$

The mapping g becomes a morphism from the partial algebra $(\mathcal{T}, \odot)$ to $(Y, *)$:

$g : \mathcal{T} \longrightarrow Y$
$g((x, u, y) \odot (y, v, z)) = g(x, u, y) * g(y, v, z)$

The values of the mapping g can be obtained by a bottom-up method. In order to describe this method we shall consider the path

$$d = ([Emily, Helen, Ann, Peter], [a_1, a_2, a_2])$$

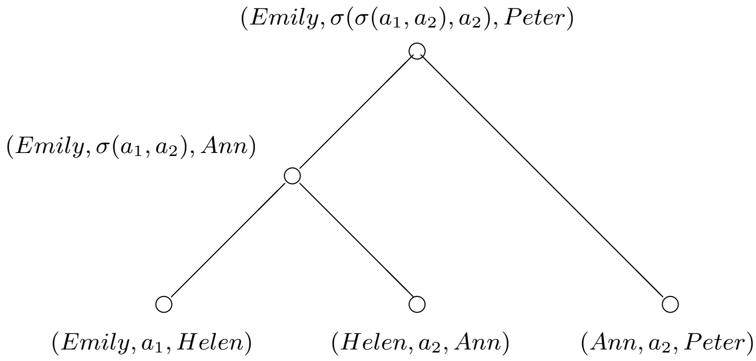


Fig. 2. The syntactic tree

in G. We consider the tuples $(Emily, a_1, Helen)$, $(Helen, a_2, Ann)$ and $(Ann, a_2, Peter)$ and we dispose these elements on the leaves of a tree as in figure 2.

Taking the image by the mapping g and its extension, we obtain the semantic tree from figure 3.

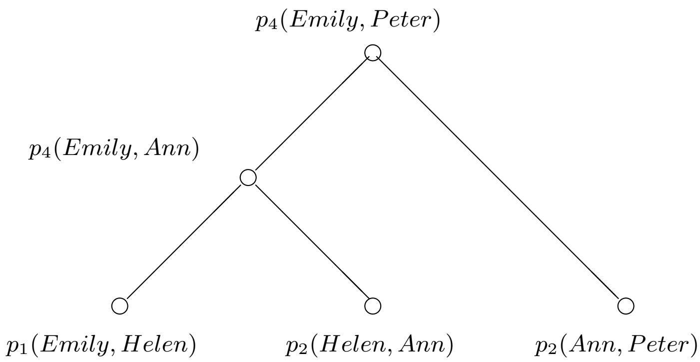


Fig. 3. The semantic tree

In conclusion, the result of the deduction process corresponding to the label $\sigma(\sigma(a_1, a_2), a_2)$ for the pair $(Emily, Peter)$ is *Emily is the niece of Peter*. The same conclusion for the deduction process is obtained if we consider the label $\sigma(a_1, \sigma(a_2, a_2))$.

We shall remark that the semantic computation is not possible for any path in G. Really, if we consider the paths $([Emily, Helen, Ann, Emily], [a_1, a_2, a_3])$ then the syntactic computation can not be realized: we can build the node $(Emily, \sigma(a_1, a_2), Ann)$, but we can not obtain the node

$$(Emily, \sigma(\sigma(a_1, a_2), a_3), Emily)$$

This is due to the fact that $\sigma(\sigma(a_1, a_2), a_3) \notin L$. Although we try to obtain the syntactic tree corresponding to the label $\sigma(a_1, \sigma(a_2, a_3))$ we find that this computation is not possible because $\sigma(a_1, \sigma(a_2, a_3)) \notin L$.

References

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