Semantic Schemas extend Semantic Networks

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Abstract. In this paper the concept of deduction in a semantic network is reconsidered in view of a new concept introduced in [5], that of semantic schema. We give an intuitive description of the deduction process in a semantic network and in addition, the deduction by confluent paths is presented. We show that a semantic schema can work as a semantic network.
Keywords: knowledge representation, confluent paths, semantic network, semantic schema
AMS Subject Classification: 68T30, 68T35

1 Semantic networks: an intuitive description

In general a semantic network is a graph structure which uses its nodes to represent concepts and its arcs to represent relations among concepts. We remark that such a structure represents the relationships between the concepts in some specific domain of knowledge. There are different kinds of relationships that are represented in a semantic network. The most common kinds are the relationships *ako*, *isa* and *has*. The abbreviations *ako* and *isa* mean *a kind of* and *is a* respectively. For example, Figure 1 contains a semantic network representing the following knowledge piece KP_1: *Bob is a bird. Every bird is a kind of animal. Every bird has wings. Every animal is alive.*

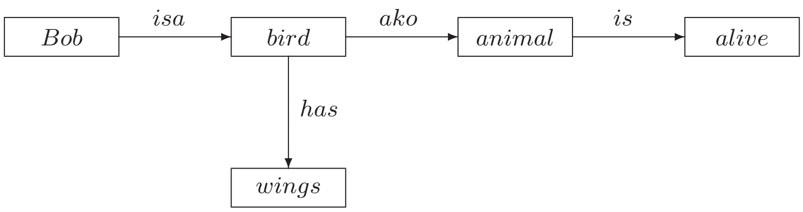

Fig. 1. Semantic network for KP_1

A semantic network can be used not only to represent knowledge but also some reasoning can be performed. In order to obtain a conclusion two nodes n

and m must be specified and a path from n to m is searched. Step by step two consecutive labels of the path are combined and the result replaces these labels. Finally only one label is obtained and this label specifies some property linking the initial and the final nodes of the path. For example, if we compose isa and has we obtain has. The conclusion obtained from the path $([Bob, bird, wings], [isa, has])$ is the sentence *Bob has wings*. We observe the *output* is a sentence in a natural language.

In this paper we consider that a semantic network has the following features:

- The arc labels are arbitrary elements, not only those specified above
- The reasoning process is based on the concept of path. Because two nodes can be connected by several arcs, a path is viewed as a pair $([x_1, \ldots, x_{n+1}], [a_1, \ldots, a_n])$, where $x_1, \ldots, x_{n+1}$ are nodes and $a_1, \ldots, a_n$ are labels such that for every $i \in \{1, \ldots, n\}$ there is an arc labelled by a_i such that x_i is the initial node and x_{i+1} is the final node of the corresponding arc.
- The conclusion of the reasoning is a sentence in a natural language.

In addition we shall consider a special case of the reasoning process, which can be described shortly as follows. Suppose that x and y are two nodes such that there is a node z such that x and y are the initial nodes of two paths and z is the final arc. In order to exemplify this situation we consider the following knowledge piece KP_2: *Bob is the son of Helen and George is the son of Peter. Peter is the brother of Susan and Helen is the sister of Susan.*

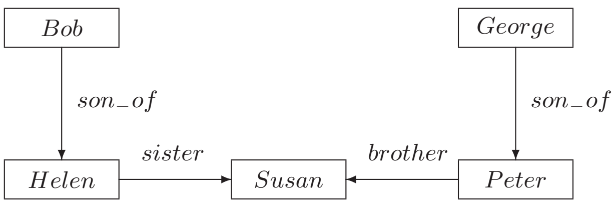

Fig. 2. Semantic network for KP_2

We say that two distinct paths $([x_1, \ldots, x_{n+1}], [a_1, \ldots, a_n])$ and $([y_1, \ldots, y_{k+1}], [b_1, \ldots, b_k])$ are *confluent* if $x_{n+1} = y_{k+1}$. For example, in Figure 2 the paths

$$[Bob, Helen, Susan], [son\_of, sister]$$

$$[George, Peter, Susan], [son\_of, brother]$$

are confluent paths. The confluence node is $Susan$. The confluent paths allow to perform additional deduction. For example, from the above representation we can deduce that *Bob is George's cousin*, *Helen is Peter's sister* and so on.

2 Semantic Schemas

The concept of *semantic schema* was introduced in [5]. This is an abstract concept which implies a *formal computation* and an *evaluation process*.

We consider the finite and nonempty sets X and A_0. Let θ be a symbol for a binary algebraic operation. We denote by $\overline{A}_0$ the Peano θ–algebra generated by A_0, therefore $\overline{A}_0 = \bigcup_{n\geq 0} M_n$ where M_n are defined recursively as follows ([1]):

$$\begin{cases} M_0 = A_0 \\ M_{n+1} = M_n \cup \{\ \theta(u,v)\ \mid\ u,v \in M_n\}, \quad n \geq 0 \end{cases}$$

Definition 1. *Let A be a set such that $A_0 \subseteq A \subseteq \overline{A}_0$ and a nonempty set $R \subseteq X \times A \times X$. We say that R and A are θ-**compatible** if the following three conditions are satisfied:*

(1.1) If $(x, \theta(u,v), y) \in R$ then there is $z \in X$ such that $(x,u,z) \in R$ and $(z,v,y) \in R$
(1.2) Let be $\theta(u,v) \in A$. For all $x,y,z \in X$ if $(x,u,z) \in R$ and $(z,v,y) \in R$ then $(x,\theta(u,v),y) \in R$
(1.3) $pr_2 R = A$

where $pr_2 R = \{\alpha \mid \exists x,y \in X : (x,\alpha,y) \in R\}$.

Definition 2. *A* **semantic schema** *is a system:*

$$\mathcal{S} = (X, A_0, A, R, \theta, \sigma, h)$$

where:

- *X is a finite nonempty set*
- *θ and σ are symbols of arity 2*
- *A_0 is a finite set and A is such that $A_0 \subseteq A \subseteq \overline{A}_0$, where $\overline{A}_0$ is the Peano θ-algebra generated by A_0*
- *$R \subseteq X \times A \times X$ is a nonempty set, such that R and A are θ-compatible*
- *h is a function symbol of arity 1*

For a given semantic schema $\mathcal{S} = (X, A_0, A, R, \theta, \sigma, h)$ we consider the following set:

$$M = \{h(x,a,y) \quad | \quad (x,a,y) \in R, a \in A_0\}$$

and denote by $\mathcal{H}$ the Peano σ-algebra generated by M ([1]).

We observe that in Definition 2 the symbol h is considered a symbol of arity 1, whereas in description of the set M the symbol h seems to be a symbol of arity 3. The contradiction can be explained by the fact that (x,a,y) is an element of R and we use the notation $h(x,a,y)$ instead of $h((x,a,y))$.

We denote by Z the alphabet including the symbol σ, the elements of X, the elements of A, the left and right parentheses, the symbol h and comma. We denote by Z^* the set of all words over Z. As in the case of a rewriting system we define two rewriting rules in the next definition.

Definition 3. *Let be* $w_1, w_2 \in Z^*$.

- *If* $a \in A_0$ *and* $(x, a, y) \in R$ *then*

$$w_1(x, a, y)w_2 \Rightarrow w_1 h(x, a, y)w_2$$

- *Let be* $(x, \theta(u, v), y) \in R$. *If* $(x, u, z) \in R$ *and* $(z, v, y) \in R$ *then*

$$w_1(x, \theta(u, v), y)w_2 \Rightarrow w_1 \sigma((x, u, z), (z, v, y))w_2$$

For our purpose it is convenient to use the reflexive and transitive closure of the relation introduced in this definition. We denote this closure by $\Rightarrow^*$.

Definition 4. *Let* $\mathcal{S} = (X, A_0, A, R, \theta, \sigma, h)$ *be a semantic schema. The* **mapping generated** *by* $\mathcal{S}$ *is the mapping*

$$\mathcal{G}_{\mathcal{S}} : R \longrightarrow 2^{\mathcal{H}}$$

defined as follows:

- $\mathcal{G}_{\mathcal{S}}(x, a, y) = \{h(x, a, y)\}$ *for* $a \in A_0$
- $\mathcal{G}_{\mathcal{S}}(x, \theta(u, v), y) = \{w \in \mathcal{H} \mid (x, \theta(u, v), y) \Rightarrow^* w\}$

We consider a finite set Ob such that $Card(X) = Card(Ob)$ and a *bijective function*

$$ob : X \longrightarrow Ob$$

For every $u \in \overline{A}_0$ we shall denote by I_u some binary relation on Ob, which is built by means of R and is stated in the next definition.

Definition 5. *For a given* $u \in \overline{A}_0$ *we define the relation* $I_u \subseteq Ob \times Ob$ *as follows:*

$$(ob(x), ob(y)) \in I_u \quad \text{iff} \quad (x, u, y) \in R$$

We specify the following properties which are proved in [5]:

- If $u \in \overline{A}_0 \setminus A$ then $I_u = \emptyset$. If $u \in A$ then $I_u \neq \emptyset$.
- Let be $\theta(u, v) \in A$. Then $I_u \circ I_v \neq \emptyset$ and $I_{\theta(u,v)} = I_u \circ I_v$, where $\circ$ represents the classical binary operation between two binary relations.

Definition 6. *Let* $\mathcal{S} = (X, A_0, A, R, \theta, \sigma, h)$ *be a semantic schema. An* **interpretation** $\mathcal{I}$ *of* $\mathcal{S}$ *is a system*

$$\mathcal{I} = (Ob, ob, Y, J_\sigma, J_h)$$

where

- Ob *is a finite set of elements which are called* **the objects** *of the interpretation.*
- $ob : X \longrightarrow Ob$ *is a bijective function*

- *Y is a nonempty set of elements which are called the* **output elements** *of the interpretation; the set Y is named the* **semantic space** *of $\mathcal{I}$*
- *$J_\sigma : Y \times Y \longrightarrow Y$ is a binary algebraic (partial) operation on Y*
- *$J_h : ELEM_\mathcal{I} \longrightarrow Y$ is a function, where*

$$ELEM_\mathcal{I} = \{(ob(x), I_a, ob(y)) \mid (x, a, y) \in R_0\}$$

where $R_0 = R \cap (X \times A_0 \times X)$

The following two computations can be performed for some semantic schema $\mathcal{S}$ and a given interpretation $\mathcal{I}$:

1) A **formal computation**, which can be stated as follows: given $(x, u, y) \in R$ find the formal entity $\mathcal{G}_\mathcal{S}(x, u, y)$ from $2^\mathcal{H}$
2) An **evaluation computation**, which can be described by the following three steps:

- Define the mapping:
$$J_{\sigma,h} : \mathcal{H} \longrightarrow Y$$
as follows:
$$\begin{cases} J_{\sigma,h}(h(x, a, y)) = J_h(ob(x), I_a, ob(y)) \text{ if } a \in A_0 \\ \\ J_{\sigma,h}(\sigma(u, v)) = J_\sigma(J_{\sigma,h}(u), J_{\sigma,h}(v)) \end{cases}$$
- For every $x, y \in X$ denote
$$AR(x, y) = \{u \in A \mid (x, u, y) \in R\}$$
Based on the mapping $\mathcal{G}_\mathcal{S} : R \longrightarrow 2^\mathcal{H}$ the following **evaluation mapping** is obtained:
$$Eval_\mathcal{I} : R \longrightarrow 2^Y$$
$$Eval_\mathcal{I}(r) = \bigcup_{t \in \mathcal{G}_\mathcal{S}(r)} \{J_{\sigma,h}(t)\}$$
- Define the **output mapping**
$$Out_\mathcal{I} : X \times X \longrightarrow 2^Y$$
$$Out_\mathcal{I}(x, y) = \bigcup_{u \in AR(x,y)} Eval_\mathcal{I}(x, u, y)$$

3 Semantic schemas extend semantic networks

In this section we show the manner in which the deduction process performed in an arbitrary given semantic network can be modeled by means of a semantic schema chosen correspondingly. We consider the semantic schema $\mathcal{S} = (X, A_0, A, R, \theta, \sigma, h)$, where

- $X = \{x_1, x_2, x_3, x_4, x_5\}$
- $A_0 = \{a_1, a_2, a_3\}$
- $A = A_0 \cup \{\theta(a_1, a_2), \theta(a_1, a_3)\}$
- $R_0 = \{(x_1, a_1, x_2), (x_2, a_2, x_3), (x_5, a_1, x_4), (x_4, a_3, x_3)\}$
- $R = R_0 \cup \{(x_1, \theta(a_1, a_2), x_3), (x_5, \theta(a_1, a_3), x_3)\}$

The conditions specified in Definition 1 are fulfilled.

A graphical representation of this semantic schema is given in Figure 3.

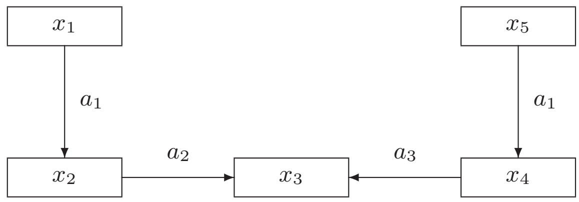

Fig. 3. Representation of a semantic schema

Let us consider the following interpretation $\mathcal{I}$ of $\mathcal{S}$:

- $Ob = \{Bob, Helen, Susan, Peter, George\}$
- $ob(x_1) = Bob$; $ob(x_2) = Helen$; $ob(x_3) = Susan$; $ob(x_4) = Peter$; $ob(x_5) = George$
- $ELEM_{\mathcal{I}} = \{(Bob, I_{a_1}, Helen), (Helen, I_{a_2}, Susan), (Peter, I_{a_3}, Susan), (George, I_{a_1}, Peter)\}$
- Y is the following set of sentences, where x and y are arbitrary elements of Ob:
  $p_1(x, y) =$ "*x is the son of y*"
  $p_2(x, y) =$ "*x is the sister of y*"
  $p_3(x, y) =$ "*x is the brother of y*"
  $p_4(x, y) =$ "*x is the nephew of y*"

The mapping J_σ is a partial one:

$J_\sigma(p_1(x, y), p_2(y, z)) = p_4(x, z)$
$J_\sigma(p_1(x, y), p_3(y, z)) = p_4(x, z)$

and the mapping J_h is defined as follows:

$J_h(Bob, I_{a_1}, Helen) = p_1(Bob, Helen)$
$J_h(Helen, I_{a_2}, Susan) = p_2(Helen, Susan)$
$J_h(George, I_{a_1}, Peter) = p_1(George, Peter)$
$J_h(Peter, I_{a_3}, Susan) = p_3(Peter, Susan)$

Now we can obtain the following formal computation:

$\mathcal{G}_\mathcal{S}(x_1, a_1, x_2) = \{h(x_1, a_1, x_2)\}$; $\mathcal{G}_\mathcal{S}(x_2, a_2, x_3) = \{h(x_2, a_2, x_3)\}$;
$\mathcal{G}_\mathcal{S}(x_5, a_1, x_4) = \{h(x_5, a_1, x_4)\}$; $\mathcal{G}_\mathcal{S}(x_4, a_3, x_3) = \{h(x_4, a_3, x_3)\}$;
$\mathcal{G}_\mathcal{S}(x_1, \theta(a_1, a_2), x_3) = \{\sigma(h(x_1, a_1, x_2), h(x_2, a_2, x_3))\}$;
$\mathcal{G}_\mathcal{S}(x_5, \theta(a_1, a_3), x_3) = \{\sigma(h(x_5, a_1, x_4), h(x_4, a_3, x_3))\}$

and the evaluation computation, which is described below:

- $J_{\sigma,h}(h(x_1, a_1, x_2)) = J_h(Bob, I_{a_1}, Helen) = p_1(Bob, Helen)$
- $J_{\sigma,h}(h(x_2, a_2, x_3)) = J_h(Helen, I_{a_2}, Susan) = p_2(Helen, Susan)$
- $J_{\sigma,h}(h(x_5, a_1, x_4)) = J_h(George, I_{a_1}, Peter) = p_1(George, Peter)$
- $J_{\sigma,h}(h(x_4, a_3, x_3)) = J_h(Peter, I_{a_3}, Susan) = p_3(Peter, Susan)$
- $J_{\sigma,h}(\sigma(h(x_1, a_1, x_2), h(x_2, a_2, x_3))) =$
  $J_\sigma(J_{\sigma,h}(h(x_1, a_1, x_2)), J_{\sigma,h}(h(x_2, a_2, x_3))) =$
  $J_\sigma(p_1(Bob, Helen), p_2(Helen, Susan)) = p_4(Bob, Susan) =$
  "*Bob is the nephew of Susan*"
- $J_{\sigma,h}(\sigma(h(x_5, a_1, x_4), h(x_4, a_3, x_3))) =$
  $J_\sigma(J_{\sigma,h}(h(x_5, a_1, x_4), J_{\sigma,h}(h(x_4, a_3, x_3))) =$
  $J_\sigma(p_1(George, Peter), p_3(Peter, Susan)) = p_4(George, Susan) =$
  "*George is the nephew of Susan*"

We obtain immediately,

$Eval_\mathcal{I}(x_1, \theta(a_1, a_2), x_3) = \{J_{\sigma,h}(\sigma(h(x_1, a_1, x_2), h(x_2, a_2, x_3)))\} =$
{ *"Bob is the nephew of Susan"* }
$Eval_\mathcal{I}(x_5, \theta(a_1, a_3), x_3) = \{J_{\sigma,h}(\sigma(h(x_5, a_1, x_4), h(x_4, a_3, x_3)))\} =$
{ *"George is the nephew of Susan"* }

and obviously

$Out_\mathcal{I}(x_1, x_3) = Eval_\mathcal{I}(x_1, \theta(a_1, a_2), x_3)$
$Out_\mathcal{I}(x_5, x_3) = Eval_\mathcal{I}(x_1, \theta(a_1, a_2), x_3)$

Remark 1. In order to include the confluent paths, a new symbol of arity 2 must be introduced in Definition 2.

4 Conclusions and future work

The aim of this paper is to show that the deduction performed in a semantic network can be obtained in a corresponding semantic schema. This process is obvious if we compare Figure 2 with Figure 3. In Figure 3 we have the same general structure as in Figure 2, but in Figure 3 the nodes are abstract nodes and the relations are abstract relations. As a matter of fact, by an interpretation a semantic schema gives a semantic network. Using distinct interpretations, from the same semantic schema we can obtain distinct semantic networks. An interesting problem is to obtain a comparative study for semantic schemas and labeled stratified graphs concerning the generative power of these mechanisms.

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