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Collaborations between distinguished representatives for labelled stratified graphs

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Abstract. The concept of labelled stratified graph (LSG) was introduced in [3] in connection with that of knowledge base with output, but a LSG can be used also in other contexts ([4]). An algebraic mechanism based on Peano algebras and morphisms of partial algebra was obtained so that the information represented in a labelled graph is processed in an algebraic manner. The set $Strat(G)$ of all $LSGs$ over a labelled graph G is divided into equivalence classes and each class contains an unique element called *distinguished representative* ([4]). In this paper we present the manner in which two distinguished representatives can be integrated in a common environment such that not only each of them can be uniquely identified but also a collaboration between them can be obtained. Some open problems are enumerated.

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1. Introduction

By *a labelled graph* we understand a tuple $G = (S, L_0, T_0, f_0)$, where S is a finite set of nodes, L_0 is a set of elements named *labels*, T_0 is a set of binary relations on S and $f_0 : L_0 \longrightarrow T_0$ is a surjective function. If each element of S is represented by a rectangle specifying the corresponding node then a labelled graph can be specified by some graphical representation. In order to obtain this representation we draw an arc from $x_1 \in S$ to $x_2 \in S$ and this arc is labelled by $a \in L_0$ if $(x_1, x_2) \in f_0(a)$. In figure 1 we show this case.

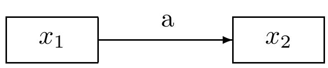

Figure 1. A labelled arc

The concept of Labelled Stratified Graph (shortly, LSG) was introduced in [3] in connection with that of knowledge base with output. A labelled stratified graph is built over some labelled graph G. In order to present this notion the following concepts are used:

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1) We consider a symbol σ of arity 2 and define recursively

$$\begin{cases} B_0 = L_0 \\ B_{n+1} = B_n \cup \{\sigma(x_1, x_2) \mid (x_1, x_2) \in B_n \times B_n\}, n \geq 0 \end{cases} \tag{1}$$

We take $B = \bigcup_{n\geq 0} B_n$. As an algebraic structure, B gives the support set of a *Peano algebra* generated by L_0 ([1]).

2) By $Initial(L_0)$ we denote some collection of subsets of B. Namely, we say that $L \in Initial(L_0)$ if the following conditions are fulfilled:

- $L_0 \subseteq L \subseteq B$
- if $\sigma(a, b) \in L$ then $a \in L$ and $b \in L$

3) If $L \in Initial(L_0)$ then the pair $(L, \{\sigma_L\})$, where

- $dom(\sigma_L) = \{(x, y) \in L \times L \mid \sigma(x, y) \in L\}$
- $\sigma_L(x, y) = \sigma(x, y)$ for every $(x, y) \in dom(\sigma_L)$

becomes a *partial algebra*.

Working with partial mappings we relieve the fact that $dom(f)$ represents the definition domain of f and we shall denote by $g \prec f$ the property "g is a *restriction* of f":

$dom(g) \subseteq dom(f)$

if $x \in dom(g)$ then $g(x) = f(x)$

We define the mapping $prod_S : dom(prod_S) \longrightarrow 2^{S\times S}$ as follows:

$dom(prod_S) = \{(\rho_1, \rho_2) \in 2^{S\times S} \times 2^{S\times S} \mid \rho_1 \circ \rho_2 \neq \emptyset\}$

$prod_S(\rho_1, \rho_2) = \rho_1 \circ \rho_2$

where $\circ$ designates the product operation between two binary relations.

We denote by $R(prod_S)$ the set of all the restrictions of the mapping $prod_S$:

$$R(prod_S) = \{u \mid u \prec prod_S\}$$

If u is an element of $R(prod_S)$ then $Cl_u(T_0)$ denotes the *closure* of T_0 in the partial algebra $(2^{S\times S}, \{u\})$. This is the smallest subset Q of $2^{S\times S}$ such that $T_0 \subseteq Q$ and Q is closed under u. It is known that this is the union $\bigcup_{n\geq 0} X_n$, where

$$\begin{cases} X_0 = T_0 \\ X_{n+1} = X_n \cup \{u(\rho_1, \rho_2) \mid (\rho_1, \rho_2) \in dom(u) \cap (X_n \times X_n)\}, n \geq 0 \end{cases} \tag{2}$$

It is known that $X_n = X_{n+1}$ for some natural number n and thus $Cl_u(T_0) = \bigcup_{k=0}^{n} X_k$ ([2]).

Definition 1.1. *([2], [3], [4]) A labelled stratified graph $\mathcal{G}$ over G is a tuple (G, L, T, u, f) where*

- $G = (S, L_0, T_0, f_0)$ *is a labelled graph*
- $L \in Initial(L_0)$
- $u \in R(prod_S)$ *and* $T = Cl_u(T_0)$
- $f : (L, \{\sigma_L\}) \longrightarrow (2^{S\times S}, \{u\})$ *is a morphism of partial algebras such that* $f_0 \prec f$, $f(L) = T$ *and if* $(f(x), f(y)) \in dom(u)$ *then* $(x, y) \in dom(\sigma_L)$

For every $\alpha \in L$ we define $trace(\alpha)$ as follows:

(1) if $\alpha \in L_0$ then $trace(\alpha) = (\alpha)$

(2) if $\alpha = \sigma(u, v)$ then $trace(\alpha) = (p, q)$, where

$$trace(u) = (p),\ trace(v) = (q)$$

In [2] we show that for every labelled graph G there exists a labelled stratified graph. In other words, if we denote by $Strat(G)$ the set of all labelled stratified graphs over G then $Strat(G) \neq \emptyset$. Moreover, if $G = (S, L_0, T_0, f_0)$ is a given labelled graph then G and a mapping $u \in R(prod_S)$ define uniquely the elements L, T, f such that $(G, L, T, u, f) \in Strat(G)$ (see Proposition 2 and its corollary in [4]). This results motivate the notation $\mathcal{G}(G, u)$ used for the element generated by u. In order to obtain this element we can apply the following algorithm ([2]):

Algorithm $LSG(G; u)$

- Take a labelled graph $G = (S, L_0, T_0, f_0)$
- Take $u \in R(prod_S)$
- Compute $T = Cl_u(T_0)$
- Take $\{B_n\}_{n\geq 0}$ as in (1)
- Define recursively for every natural number $n \geq 0$:

$D_{n+1} = \{\sigma(p, q) \in B_{n+1} \setminus B_n \mid p, q \in dom(f_n), (f_n(p), f_n(q)) \in dom(u)\}$
$dom(f_{n+1}) = dom(f_n) \cup D_{n+1}$

$$f_{n+1}(x) = \begin{cases} f_n(x) & \text{if} \quad x \in dom(f_n) \\ \\ u(f_n(p), f_n(q)) & \text{if} \quad x = \sigma(p, q) \in D_{n+1} \end{cases}$$

- Define the mapping $f : dom(f) \longrightarrow T$ as follows:

$dom(f) = \bigcup_{n\in N} dom(f_n) = L_0 \cup \bigcup_{k\geq 1} D_k$

$$f(x) = \begin{cases} f_0(x) & \text{if} \quad x \in L_0 \\ \\ f_k(x) & \text{if} \quad x \in D_k \end{cases}$$

- Take $L = dom(f)$
- Obtain $(G, L, T, u, f) \in Strat(G)$

End of Algorithm

Remark 1.1. *Because some confusion can appear in the notation $Cl_u(T_0)$, in the remainder of this paper we denote $H_G(u) = Cl_u(T_0)$ in order to highlight the fact that the closure is taken with respect to G. This specification is necessary because distinct labelled graphs will be used.*

We mention now that two distinct mappings can generate the same labelled stratified graph ([4]). In order to avoid this situation and to identify a set of mappings such that different mappings generate different labelled stratified graphs we introduced in [4] the operator θ_G and the set $MGE(G)$ as in the next definition:

Definition 1.2. *([4]) Let $G = (S, L_0, T_0, f_0)$ be a labelled graph. We define the operator*

$$\theta_G : R(prod_S) \longrightarrow R(prod_S)$$

taking $\theta_G(u) \prec u$ such that $dom(\theta_G(u)) = (H_G(u) \times H_G(u)) \cap dom(u)$. We denote by $MGE(G)$ the image of the set $R(prod_S)$ by θ_G, that is, $MGE(G) = \theta_G(R(prod_S))$.

In order to obtain in a bijective manner all the $LSGs$ over some labelled graph G it suffices to consider only the elements of the set $MGE(G)$ ([4]).

Let f and g be two mappings such that $f(x) = g(x)$ for every $x \in dom(f) \cap dom(g)$. We define the mapping $f \vee g$ as follows:

$$dom(f \vee g) = dom(f) \cup dom(g)$$

$$(f \vee g)(x) = \begin{cases} f(x) \text{ if } x \in dom(f) \\ \\ g(x) \text{ if } x \in dom(g) \end{cases} \tag{3}$$

If $G = (S, L_0, T_0, f_0)$ is a labelled graph then a path in G is a pair

$$d = ([x_1, \ldots, x_{n+1}], [e_1, \ldots, e_n])$$

such that $x_1, \ldots, x_{n+1} \in S$, $e_1, \ldots, e_n \in L_0$ and $(x_i, x_{i+1}) \in f_0(e_i)$ for $i \in \{1, \ldots, n\}$. If this is the case, then d is called a path from x_1 to x_{n+1} labelled by the sequence $(e_1, \ldots, e_n)$.

In [4] we introduced an equivalence relation on the set $Strat(G)$ of all labelled stratified graphs over G. We introduced also a partial order on the set $Strat(G)$. We proved that there is the least element for each equivalence class $[\mathcal{G}(G,u)]$ (see Proposition 31, [4]). This is $\mathcal{G}(G, \theta_G(u))$ and it is called *the distinguished representative* of the class $[\mathcal{G}(G,u)]$. Because this is a special element in the equivalence class generated by u, it is denoted by $DR(u)$. On the other hand, $(R(prod_S), \prec)$ is a partial ordered set such that $sup\{u,v\}$ exists and $sup\{u,v\} = u \vee v$. Thus, if $u, v \in R(prod_S)$ then we can consider the distinguished representatives $DR(u)$, $DR(v)$ and $DR(u \vee v)$. In the next section we shall refer to this representatives.

2. Collaboration between distinguished representatives

In the last part of [4] we presented an application whose provenance comes from a fusion action of two companies C_1 and C_2. The services accomplished by C_i are described by a labelled graph G_i $(i = 1, 2)$. In order to obtain a satisfactory description we have to consider the mappings $u = \theta_{G_1}(prod_S)$, $v = \theta_{G_2}(prod_S)$ and $u \vee v$. We are interested to integrate $DR(u)$ and $DR(v)$ in $DR(u \vee v)$. Some collaboration between $DR(u)$ and $DR(v)$ will be obtained in a natural manner but in order to impose additional collaboration we have to complete the graph $DR(u \vee v)$. The description of these operations is the aim of this section.

In order to fix the ideas we shall consider the labelled graphs G_1 from Figure 2 and G_2 from Figure 3.

We shall take:

1) $G_1 = (S, L_0^{(1)}, T_0^{(1)}, f_0^{(1)})$ where
- $S = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
- $L_0^{(1)} = \{a_1, b_1, c_1\}$
- $T_0^{(1)} = \{\rho_1, \rho_2, \rho_3\}$, where
  $\rho_1 = \{(x_1, x_2), (x_3, x_4)\}$, $\rho_2 = \{(x_2, x_3), (x_5, x_6)\}$, $\rho_3 = \{(x_6, x_7)\}$
- $f_0(a_1) = \rho_1$, $f_0(b_1) = \rho_2$, $f_0(c_1) = \rho_3$

Taking $u = \theta_{G_1}(prod_S)$ we obtain Table 1, where x denotes the fact that the corresponding relations can not be composed by the product operation and

$\mu_1 = \{(x_1, x_3)\}$, $\mu_2 = \{(x_2, x_4)\}$, $\mu_3 = \{(x_5, x_7)\}$, $\mu_4 = \{(x_1, x_4)\}$

Computing the components of $\mathcal{G}(G_1, u) = (G_1, L^{(1)}, T^{(1)}), f^{(1)}$ we obtain:
- $D_0^{(1)} = L_0^{(1)} = \{a_1/\rho_1, b_1/\rho_2, c_1/\rho_3\}$
- $D_1^{(1)} = \{\sigma(a_1, b_1)/\mu_1, \sigma(b_1, a_1)/\mu_2, \sigma(b_1, c_1)/\mu_3\}$
- $D_2^{(1)} = \{\sigma(a_1, \sigma(b_1, a_1))/\mu_4, \sigma(\sigma(a_1, b_1), a_1)/\mu_4\}$
- $L^{(1)} = D_0^{(1)} \cup D_1^{(1)} \cup D_2^{(1)}$
- $T^{(1)} = \{\rho_1, \rho_2, \rho_3, \mu_1, \mu_2, \mu_3, \mu_4\}$

where we denoted by α/ρ the property $f^{(1)}(\alpha) = \rho$.

2) $G_2 = (S, L_0^{(2)}, T_0^{(2)}, f_0^{(2)})$ where
- $S = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
- $L_0^{(2)} = \{a_2, b_2, c_2\}$
- $T_0^{(2)} = \{\rho_1, \rho_2, \rho_4\}$, where $\rho_4 = \{(x_4, x_5)\}$
- $f_0(a_2) = \rho_1$, $f_0(b_2) = \rho_2$, $f_0(c_2) = \rho_4$

Taking $v = \theta_{G_2}(prod_S)$ we obtain Table 2, where

u	ρ_1	ρ_2	ρ_3	μ_1	μ_2	μ_3	μ_4
ρ_1	x	μ_1	x	x	μ_4	x	x
ρ_2	μ_2	x	μ_3	x	x	x	x
ρ_3	x	x	x	x	x	x	x
μ_1	μ_4	x	x	x	x	x	x
μ_2	x	x	x	x	x	x	x
μ_3	x	x	x	x	x	x	x
μ_4	x	x	x	x	x	x	x

TABLE 1. The mapping u for G_1

$\mu_5 = \{(x_1, x_5)\}$; $\mu_6 = \{(x_2, x_5)\}$; $\mu_7 = \{(x_2, x_6)\}$
$\mu_8 = \{(x_3, x_6)\}$; $\mu_{11} = \{(x_1, x_6)\}$

If we compute the components of $\mathcal{G}(G_2, v) = (G_2, L^{(2)}, T^{(2)}, v, f^{(2)})$ we obtain:

- $D_0^{(2)} = L_0^{(2)} = \{a_2/\rho_1, b_2/\rho_2, c_2/\rho_4\}$
- $D_1^{(2)} = \{\sigma(a_2, b_2)/\mu_1, \sigma(a_2, c_2)/\nu_1, \sigma(b_2, a_2)/\mu_2, \sigma(c_2, b_2)/\nu_2\}$
- $D_2^{(2)} = \{\sigma(a_2, \sigma(b_2, a_2))/\mu_4, \sigma(a_2, \sigma(c_2, b_2))/\mu_8, \sigma(b_2, \sigma(a_2, c_2))/\mu_6,$
  $\sigma(\sigma(a_2, b_2), a_2)/\mu_4,\ \sigma(\sigma(a_2, b_2), \sigma(a_2, c_2))/\mu_5, \sigma(\sigma(b_2, a_2), c_2)/\mu_6,$
  $\sigma(\sigma(b_2, a_2), \sigma(c_2, b_2))/\mu_7, \sigma(\sigma(a_2, c_2), b_2)/\mu_8\}$
- $D_3^{(2)} = \{\sigma(a_2, \sigma(b_2, \sigma(a_2, c_2)))/\mu_5, \sigma(a_2, \sigma(\sigma(b_2, a_2), c_2))/\mu_5,$
  $\sigma(a_2, \sigma(\sigma(b_2, a_2), \sigma(c_2, b_2)))/\mu_{11}, \sigma(b_2, \sigma(a_2, \sigma(c_2, b_2)))/\mu_7,$
  $\sigma(b_2, \sigma(\sigma(a_2, c_2), b_2))/\mu_7,\ \sigma(\sigma(a_2, b_2), \sigma(\sigma(a_2, c_2), b_2))/\mu_{11},$
  $\sigma(\sigma(a_2, b_2), \sigma(a_2, \sigma(c_2, b_2)))/\mu_{11}, \sigma(\sigma(a_2, \sigma(b_2, a_2)), c_2)/\mu_5,$
  $\sigma(\sigma(\sigma(a_2, b_2), a_2), c_2)/\mu_5,\ \sigma(\sigma(a_2, \sigma(b_2, a_2)), \sigma(c_2, b_2))/\mu_{11},$
  $\sigma(\sigma(\sigma(a_2, b_2), a_2), \sigma(c_2, b_2))/\mu_{11}, \sigma(\sigma(\sigma(a_2, b_2), \sigma(a_2, c_2)), b_2)/\mu_{11},$
  $\sigma(\sigma(b_2, \sigma(a_2, c_2)), b_2)/\mu_7, \sigma(\sigma(\sigma(b_2, a_2), c_2), b_2)/\mu_7\}$
- $L^{(2)} = D_0^{(2)} \cup D_1^{(2)} \cup D_2^{(2)} \cup D_3^{(2)}$
- $T^{(2)} = T_0^{(2)} \cup \{\mu_1, \mu_2, \nu_1, \nu_2, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8, \mu_{11}\}$

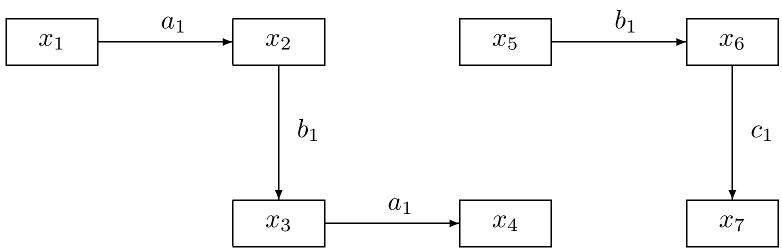

FIGURE 2. Labelled graph G_1

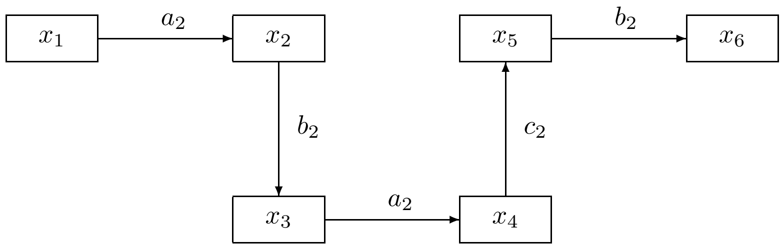

FIGURE 3. Labelled graph G_2

v	ρ_1	ρ_2	ρ_4	μ_1	μ_2	ν_1	ν_2	μ_4	μ_5	μ_6	μ_7	μ_8	μ_{11}
ρ_1	x	μ_1	ν_1	x	μ_4	x	μ_8	x	x	μ_5	μ_{11}	x	x
ρ_2	μ_2	x	x	x	x	μ_6	x	x	x	x	x	μ_7	x
ρ_4	x	ν_2	x	x	x	x	x	x	x	x	x	x	x
μ_1	μ_4	x	x	x	x	μ_5	x	x	x	x	x	μ_{11}	x
μ_2	x	x	μ_6	x	x	x	μ_7	x	x	x	x	x	x
ν_1	x	μ_8	x	x	x	x	x	x	x	x	x	x	x
ν_2	x	x	x	x	x	x	x	x	x	x	x	x	x
μ_4	x	x	μ_5	x	x	x	μ_{11}	x	x	x	x	x	x
μ_5	x	μ_{11}	x	x	x	x	x	x	x	x	x	x	x
μ_6	x	μ_7	x	x	x	x	x	x	x	x	x	x	x
μ_7	x	x	x	x	x	x	x	x	x	x	x	x	x
μ_8	x	x	x	x	x	x	x	x	x	x	x	x	x
μ_{11}	x	x	x	x	x	x	x	x	x	x	x	x	x

TABLE 2. The mapping v for G_2

$u \vee v$	ρ_1	ρ_2	ρ_3	ρ_4	μ_1	μ_2	μ_3	ν_1	ν_2	μ_4	μ_5	μ_6	μ_7	μ_8	μ_{11}
ρ_1	x	μ_1	x	ν_1	x	μ_4	x	x	μ_8	x	x	μ_5	μ_{11}	x	x
ρ_2	μ_2	x	μ_3	x	x	x	x	μ_6	x	x	x	x	x	μ_7	x
ρ_3	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
ρ_4	x	ν_2	x	x	x	x		x	x	x	x	x	x	x	x
μ_1	μ_4	x	x	x	x	x	x	μ_5	x	x	x	x	x	μ_{11}	x
μ_2	x	x	x	μ_6	x	x	x	x	μ_7	x	x	x	x	x	x
μ_3	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
ν_1	x	μ_8	x	x	x	x		x	x	x	x	x	x	x	x
ν_2	x	x		x	x	x	x	x	x	x	x	x	x	x	x
μ_4	x	x	x	μ_5	x	x	x	x	μ_{11}	x	x	x	x	x	x
μ_5	x	μ_{11}	x	x	x	x		x	x	x	x	x	x	x	x
μ_6	x	μ_7	x	x	x	x		x	x	x	x	x	x	x	x
μ_7	x	x		x	x	x	x	x	x	x	x	x	x	x	x
μ_8	x	x		x	x	x	x	x	x	x	x	x	x	x	x
μ_{11}	x	x		x	x	x	x	x	x	x	x	x	x	x	x

TABLE 3. The mapping $u \vee v$

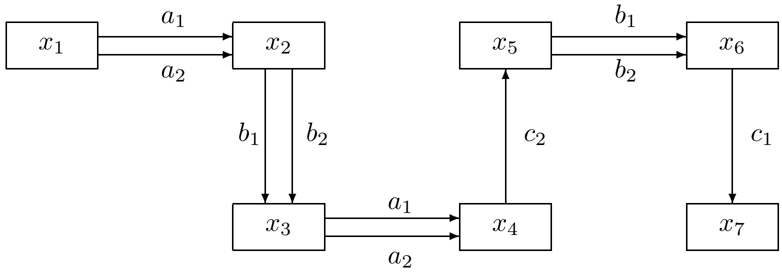

FIGURE 4. Labelled graph $G_{1\cup 2}$

Taking the union graph for G_1 and G_2 we obtain the labelled graph $G_{1\cup 2}$ represented in Figure 4.

The mapping $u \vee v$ is given in Table 3. If we compute the set $L_{u\vee v}$ of the labels for $\mathcal{G}(G_{1\cup 2}, u \vee v)$ we obtain

$$L_{u\vee v} = L^{(1)} \cup L^{(2)} \cup L_{(1,2)} \tag{4}$$

Each element $\alpha \in L_{(1,2)}$ has the property that $trace(\alpha)$ contains both elements from $L_0^{(1)}$ and elements from $L_0^{(2)}$. Thus the elements of $L_{(1,2)}$ have a structure that shows that $DR(u)$ and $DR(v)$ collaborate in $DR(u \vee v)$.

If we examine the set $L_{(1,2)}$ we find that there is a path from x_1 to x_6 because $(x_1, x_6) \in \mu_{11}$ and there is, for example, the "combined" label

$$\alpha = \sigma(\sigma(a_1, b_2), \sigma(a_1, \sigma(c_2, b_1)))$$

such that $f(\alpha) = \mu_{11}$.

In the same time, none of the binary relations ρ in Table 3 satisfies the condition $(x_1, x_7) \in \rho$. Equivalently, this means that $\mathcal{G}(G_{1\cup 2}, u \vee v)$ does not authorize the use of any path from x_1 to x_7 in $G_{1\cup 2}$. In order to benefit of such a path we have to fill in some position in Table 3. If we proceed in this manner then we obtain a completion of $\mathcal{G}(G_{1\cup 2}, u \vee v)$. For example, if in the place corresponding to the line μ_{11} and column ρ_3 we append in Table 3 the element

$$prod_S(\mu_{11}, \rho_3) = \mu_{12}$$

then the new LSG will authorize the use of the path

$$([x_1, x_1, x_3, x_4, x_5, x_6, x_7], [a_1, b_2, a_1, c_2, b_1, c_1])$$

in $G_{1\cup 2}$ because $\sigma(\alpha, c_1)$ becomes a label for this LSG.

Various completions for $\mathcal{G}(G_{1\cup 2}, u \vee v)$ can be obtained. If we fill in all the empty places in Table 3 then we take:

$prod_S(\nu_1, \mu_3) = prod_S(\mu_8, \rho_3) = \mu_9$
$prod_S(\rho_4, \mu_3) = prod_S(\nu_2, \rho_3) = \mu_{10}$
$prod_S(\mu_{11}, \rho_3) = prod_S(\mu_5, \mu_3) = \mu_{12}$
$prod_S(\mu_7, \rho_3) = prod_S(\mu_6, \mu_3) = \mu_{13}$

Let us denote by L_c the label set for this case. We can imagine the following situation appeared in an application: the nodes of a labelled graph represent localities of a county; the arcs represent variants for motor-ways; a label represents the weather state for a variant. We may be interested to convey some goods from x_1 to x_7. Suppose the conditions imposed by the quality of the goods require the use of a path containing a minimum number of symbols a_2, b_2, c_2. This problem reduces to the finding of the set

$$L_{\mu_{12}} = \{\alpha \in L_c \mid f(\alpha) = \mu_{12}, Pl(\alpha) = min\}$$

where $Pl(\alpha)$ represents the number of places from α such that each place contains a symbol a_2, b_2 or c_2.

If we compute the elements of L_c we find that for each $\alpha \in L_{\mu_{12}}$ we have

$$trace(\alpha) = (a_1, b_1, a_1, c_2, b_1, c_1)$$

We can conclude by this particular computation that there is only one authorized path satisfying the conditions imposed above and this path has the length 6.

3. Conclusions and open problems

Various properties and applications for the set $Strat(G)$ of all the labelled stratified graphs over some labelled graph G was presented in [2], [3] and [4]. In this paper we inaugurate a possible research line concerning the combination of two such structures which are built over distinct labelled graphs. Various completions of $\mathcal{G}(G, u \vee v)$ can be obtained in order to obtain a better collaboration between $\mathcal{G}(G_1, u)$ and $\mathcal{G}(G_2, v)$. We are interested to develop the following questions:

- Consider the label set $L_{(1,2)}$ from (4) and give an expression by means of which we can compute this set only by the components of $\mathcal{G}(G_1, u)$ and $\mathcal{G}(G_2, v)$. This expression will give an analytical characterization for the collaboration between $DR(u)$ and $DR(v)$ in $DR(u \vee v)$.
- Consider the concept of distinguished representative given in [4] and study the impact of the ideas presented in this paper concerning various combination of such representatives which are built over different labelled graphs.

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