# Image Synthesis from Natural Language Description 

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#### Abstract

In this paper we present another application of the labelled stratified graphs introduced in [2]. More exactly, we consider a text $T$ given in a natural language. $T$ gives a description of some world of objects. Using a recursive transition network ([1]) the information contained in $T$ is extracted. As a consequence, a labelled graph $G$ is obtained. The structure of $G$ is extended to a labelled stratified graph and applying the appropriate methods we can realize the synthesis of the image specified in $T$. The method is implemented and all the formal computations are presented. In conclusion, taking into account a text describing an image, we present an application of the labelled stratified graphs in image synthesis.


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## 1 General presentation

The concept of labelled stratified graph ( $L S G$ ) was introduced in [2] in connection with that of knowledge base with output (KBO). Various applications of this concept in travel scheduling, attribute graphs and transport problems where presented. In general, this concept is used in connection with knowledge representation. As a general presentation of this method we can say that in the first step a labelled graph $G$ is obtained by extracting the information from a given knowledge piece. In the second step a $L S G$ is built over $G$. In the last step, the algebraic mechanism offered by a $L S G$ is used to formalize the reasoning process.

In this paper the following aspects are treated:

1) We give an algorithm by means of which we are able to extract the information from the text of a knowledge piece $K P$. The information is represented by objects and the binary relations between them. Automatically, the drawing of the corresponding labelled graph $G$ is realized. In order to process the information from $K P$ a recursive transition network is used. A
$L S G$ over $G$ is defined and based on this structure an appropriate reasoning environment is obtained.
2) The above method is exemplified and the means offered by a $L S G$ are used to reconstruct the image of the world described in $K P$. We used here the term "reconstruct" for the following reason: the method can be applied in a communication from the location A to the remote location B in order to reconstruct in B the image observed in A . In this case a text description of the scene is transmitted instead of an image. The text is transmitted from A and based on this description an appropriate image is reconstructed in B.

The paper is organized as follows: in Section 2 a concise presentation of the concept of labelled stratified graph is given; in Section 3 we give a general algorithm to draw automatically the labelled graph associated to a text description given in a natural language whose syntax is represented by means of a recursive transition network; based on the concept of $L S G$ we present in Section 4 the manner in which we can synthetize the image described by a text description; Section 5 includes an implementation of the method presented in this paper; in the last section several future problems are presented.

## 2 Labelled Stratified Graphs

In this section we give a brief presentation of the concept of labelled stratified graph (shortly, $L S G$ ), which is a basic one for the next sections.

Intuitively, a labelled graph is a directed graph such that each arc contains an entity called label such that, finally, this entity carries a specific information.

Definition 1. A labelled graph is a tuple $G=\left(S, L_{0}, T_{0}, f_{0}\right)$, where

- $S$ is a finite set of elements called nodes
- $L_{0}$ is a finite set of elements named labels
- $T_{0}$ is a set of binary relations on $S$
- $f_{0}: L_{0} \longrightarrow T_{0}$ is a surjective function

Such a structure admits a graphical representation. Each element of $S$ is represented by a rectangle specifying the corresponding node. We draw an arc from $x \in S$ to $y \in S$ and this arc is labelled by $a \in L_{0}$ if $(x, y) \in f_{0}(a)$. This representation is shown in figure 1.


Fig. 1. A labelled arc

We consider a labelled graph $G=\left(S, L_{0}, T_{0}, f_{0}\right)$ and a symbol $\sigma$ of arity 2 . We define recursively

$$
\left\{\begin{array}{l}
B_{0}=L_{0}  \tag{1}\\
B_{n+1}=B_{n} \cup\left\{\sigma\left(x_{1}, x_{2}\right) \mid\left(x_{1}, x_{2}\right) \in B_{n} \times B_{n}\right\}, n \geq 0
\end{array}\right.
$$

The set $B=\bigcup_{n \geq 0} B_{n}$ gives the support set of a Peano algebra generated by $L_{0}$ ([?]).

By $\operatorname{Initial}\left(L_{0}\right)$ we denote some collection of subsets of $B$. Namely, we say that $M \in \operatorname{Initial}\left(L_{0}\right)$ if the following conditions are fulfilled:

- $L_{0} \subseteq M \subseteq B$
- if $\sigma(u, v) \in M$ then $u \in M$ and $v \in M$

We define the mapping $\operatorname{prod}_{S}: \operatorname{dom}\left(\operatorname{prod}_{S}\right) \longrightarrow 2^{S \times S}$ as follows:

$$
\begin{aligned}
& \operatorname{dom}\left(\operatorname{prod}_{S}\right)=\left\{\left(\rho_{1}, \rho_{2}\right) \in 2^{S \times S} \times 2^{S \times S} \mid \rho_{1} \circ \rho_{2} \neq \emptyset\right\} \\
& \operatorname{prod}_{S}\left(\rho_{1}, \rho_{2}\right)=\rho_{1} \circ \rho_{2}
\end{aligned}
$$

where $\rho_{1} \circ \rho_{2}$ represents the product of the binary relation $\rho_{1}$ and $\rho_{2}$ :

$$
\rho_{1} \circ \rho_{2}=\left\{(x, y) \mid \exists z:(x, z) \in \rho_{1},(z, y) \in \rho_{2}\right\}
$$

We denote by $R\left(\operatorname{prod}_{S}\right)$ the set of all the restrictions of the mapping $\operatorname{prod}_{S}$ :

$$
R\left(\operatorname{prod}_{S}\right)=\left\{u \mid u \prec \operatorname{prod}_{S}\right\}
$$

Let $u$ be an element of $R\left(\operatorname{prod}_{S}\right)$. We denote by $C l_{u}\left(T_{0}\right)$ the closure of $T_{0}$ in the partial algebra $\left(2^{S \times S},\{u\}\right)$. This is the smallest subset $Q$ of $2^{S \times S}$ such that $T_{0} \subseteq Q$ and $Q$ is closed under $u$. It is known that this is the union $\bigcup_{n \geq 0} X_{n}$, where

$$
\left\{\begin{array}{l}
X_{0}=T_{0}  \tag{2}\\
X_{n+1}=X_{n} \cup\left\{u\left(\rho_{1}, \rho_{2}\right) \mid\left(\rho_{1}, \rho_{2}\right) \in \operatorname{dom}(u) \cap\left(X_{n} \times X_{n}\right)\right\}, n \geq 0
\end{array}\right.
$$

Because $S$ is a finite set, there is $n$ such that $X_{n}=X_{n+1}$ and thus $C l_{u}\left(T_{0}\right)=$ $\bigcup_{k=0}^{n} X_{k}([3])$.

Definition 2. A labelled stratified graph $\mathcal{G}$ over $G$ is a tuple $(G, L, T, u, f)$ where

- $G=\left(S, L_{0}, T_{0}, f_{0}\right)$ is a labelled graph
- $L \in \operatorname{Initial}\left(L_{0}\right)$
- $u \in R\left(\operatorname{prod}_{S}\right)$ and $T=C l_{u}\left(T_{0}\right)$
- $f:\left(L,\left\{\sigma_{L}\right\}\right) \longrightarrow\left(2^{S \times S},\{u\}\right)$ is a morphism of partial algebras such that $f_{0} \prec f, f(L)=T$ and if $(f(x), f(y)) \in \operatorname{dom}(u)$ then $(x, y) \in \operatorname{dom}\left(\sigma_{L}\right)$


Fig. 2. The morphism condition

The last condition imposed in the definition of $L S G$ can be interpreted as follows. The mapping $f$ is a morphism of partial algebras, therefore in the diagram of figure 2 we have the following property: if we are able to go along the path $\left(L \times L, L, 2^{S \times S}\right)$ then we are able also to go along the path $\left(L \times L, 2^{S \times S} \times\right.$ $2^{S \times S}, 2^{S \times S}$ ) and we obtain the same result.

We observe now that the definition of a $L S G$ gives the converse condition: if we can go along the path ( $L \times L, 2^{S \times S} \times 2^{S \times S}, 2^{S \times S}$ ) then we can go along the path $\left(L \times L, L, 2^{S \times S}\right)$ and we obtain the same final result. Indeed, if $(f(x), f(y)) \in$ $\operatorname{dom}(u)$ then $(x, y) \in \operatorname{dom}\left(\sigma_{L}\right)$ therefore $\sigma_{L}(x, y) \in L$. Now by the morphism property we have $f\left(\sigma_{L}(x, y)\right)=u(f(x), f(y))$.

Based on these two remarks we observe that

$$
\begin{equation*}
\operatorname{dom}\left(\sigma_{L}\right)=\{(x, y) \in L \times L \mid(f(x), f(y)) \in \operatorname{dom}(u)\} \tag{3}
\end{equation*}
$$

## 3 A general algorithm for automated drawing of a labelled graph associated to a text description

In this section we will present the manner in which a knowledge piece given in a natural language (English) can be transposed in a labelled graph. To make possible such a representation we shall suppose the knowledge piece describes several objects and some binary relations between them.

In order to extract the relations contained in the input text we have to define the syntax of the language, based on which we can also verify the correctness of the text description. We choose a mechanism based on a Recursive Transition Network $(R T N)$ because the $R T N s$ are modular and very easy to maintain. They are based on Basic Transition Networks BTN. These networks give us a graphical representation of the behaviour of some kind of finite state automaton. So the BTNs can be thought of as directed labelled graphs. The nodes of the graph are the states of the automaton. Unlike $B T N s$, the $\operatorname{arcs}$ of $R T N s$ can be
labelled with a command JUMP node. The node appearing in such a command must be a start node of a subnetwork. When an arc labelled with this command is traversed, a jump will be made to the subnetwork that has as initial node the node specified in the JUMP command. This network will be traversed and finally, only when it is reached its final state, the transition will continue from the point it was made the jump. This idea of breaking up a complex network into some smaller networks, one for every syntactic category (such as sentence, noun phrase, verb phrase, etc.) helps us to obtain a better structure for our linguistic knowledge. Thus, all the information about a category are contained in a corresponding subnetwork. This facilitates any syntactical modification in the language we want to define.

After we establish the desired $R T N$, we have to determine the binary relations that are contained in a given knowledge piece $K P$. When all the relations are obtained we can draw the directed labelled graph $G=\left(S, L_{0}, T_{0}, f_{0}\right)$. The set $S$ of nodes is composed from the objects of the knowledge piece and the elements of $L_{0}$ are labels of binary relations. The elements of $T_{0}$ are the initial binary relations, specified in $K P$. The surjective mapping $f_{0}: L_{0} \longrightarrow T_{0}$ establishes the connection between labels and binary relations.

We will start the drawing of nodes with a node that has a maximum number of children and a minimum number of parents. The line of this first node will be considered the first line of the graph. The same for the column of the first node. The rest of the nodes will be drawn using the following algorithm:

1. current_node $\leftarrow$ first_node
2. current_line $\leftarrow$ first_line
3. current_column $\leftarrow$ first_column
4. WHILE (we still have not drawn all the nodes)
4.1 IF (current_node has child)
4.1.1 we will prefer that node, named newNode, that accomplishes the most conditions from below:
4.1.1.1 it is child of current_node
4.1.1.2 \{parents of newNode $\} \cap\{$ children of current_node $\}=\emptyset$
4.1.1.3 the current_node is a child of newNode
4.1.2 current_column $\leftarrow$ current_column +1
4.1.3 draw newNode on the cell (current_line, current_column)
4.1.4 the current_node will be connected with the newNode by a straight arc.
4.1.5 delete newNode from the set of children of current_node
4.1.6 current_node $\leftarrow$ newNode
4.2 ENDIF
4.3 IF (current_node does not have any child)
4.3.1 current_line $\leftarrow$ current_line +1 ;
current_column $\leftarrow$ current_column - 1
4.3.2 IF (current_node has a drawn parent)
4.3.2.1 current_node $\leftarrow$ the last drawn parent of current_node 4.3.3 ELSE
4.3.3.1 that means we have now to draw the nodes that have as children the already drawn nodes
4.3.3.2 we will choose as current_node, a node with most of the children already drawn

### 4.3.4 ENDIF

4.4 ENDIF
5. ENDWHILE

After all the nodes are drawn, we have to do the same with the arcs. We divide the arcs into two categories:

- straight arcs: these arcs are oriented from left to right and connect a node with a child found in the $I F$ sentence at line 4.1 (for example, in Figure 8 the arc from $p 2$ to $p 4$ labelled by $B+L$ )
- broken arcs: these arcs connect two nodes going around the nodes of the graph (for example, in Figure 8 the arc from one_line to $p 4$ labelled by $A$ )


## 4 From text description to image synthesis

In this section we shall present an application to reconstruct an image from a text description. We consider the following problem:

We suppose we have a remote communication line from $S$ to $R . S$ is the sender and $R$ is the receiver. Let's suppose at the node $S$ we have a chess-board of only $5 \times 5$ squares and five pawns of the same colour. We say that a pawn $P_{1}$ captures the pawn $P_{2}$ if $P_{1}$ and $P_{2}$ are aligned on the same diagonal of the board at one square distance. We suppose that in $S$ we see an arrangement of these pieces such that no pawn captures any other pawn. The problem is to transmit to $R$ a text description of the scene such that $R$ is able to reconstruct the image from $S$.

In order to solve this problem we try to use in R a text analyzer, which is able to understand the text received from S and to reconstruct the image on R . This program is a text to image converter. We suppose the following:

- We dispose of a free-error transmission.
- The description does not specify exactly the position of a pawn, that is, the use of a sentence of the form a pawn is on the line $i$ and column $j$ is forbidden.
- Each sentence describes the position of some pawn with respect to some other pawn or the position of some empty line (column).
The description that program gets as input can be seen as a knowledge piece $K P$ of some world of objects. Obviously, the knowledge piece will contain at least five objects, the pawns denoted by $P_{1}, \ldots, P_{5}$. Thus, the set $S$ of objects will include the set of pawns, $S \supseteq\left\{P_{1}, \ldots, P_{5}\right\}$.

In order to help the program to find the positions of the pawns, the knowledge piece must contain five sentences that include binary relations between pawns. Let's denote by $S e n_{1}, \ldots, S e n_{5}$ five sentences of the $K P$, where $S e n_{i}$ appears in
the input description before $\operatorname{Sen}_{i+1}, 1 \leq i \leq 4$. A good description must allow us to obtain information about the position of any pawn on the board.

Before to give a formalism of this problem we introduce the following notation: if $S e n_{k}$ gives a description of the relation between $P_{i}$ and $P_{j}$ then we note: $\operatorname{Sen}_{k}=\left(P_{i}, P_{j}\right)$. We suppose $\operatorname{Sen}_{1}=\left(P_{k_{1}}, P_{k_{2}}\right)$, $\operatorname{Sen}_{2}=\left(P_{k_{2}}, P_{k_{3}}\right)$, $\operatorname{Sen}_{3}=\left(P_{k_{3}}, P_{k_{4}}\right), \operatorname{Sen}_{4}=\left(P_{k_{4}}, P_{k_{5}}\right)$ and $\operatorname{Sen}_{5}=\left(P_{k_{5}}, P_{k_{1}}\right)$ where $1 \leq k_{i} \leq 5$ and $k_{i}$ are distinct values. In this way, the sentences $S e n_{1}, \ldots, S e n_{5}$ offer a maximum number of information about the objects $P_{1}, \ldots, P_{5}$. Thus, if we transpose the relations of these five sentences in a directed labelled graph, we obtain a cycle that permits us to get new information about any two nodes using deduction process (see Figure 3).


Fig. 3. A cycle of pawns

Each relation of these five sentences indicates the position of a pawn regarding some other pawn. Before enumerate the possible positions we make the following observation: a square is uniquely defined by its coordinates on the board, the pair (line, column) where $1 \leq$ line, column $\leq 5$. Thus, instead of saying that the pawn $P$ is on the line $l$ and column $c$, we will say that the square of the pawn $P$ is $(l, c)$.

Finally, the possible positions of a pawn are:

- above: a pawn $P_{m}$ is above another pawn $P_{n}$ if the square of $P_{m}$ is $(\mathrm{i}, \mathrm{j})$ and the square of $P_{n}$ is $(\mathrm{k}, \mathrm{j})$ with $k<i$.
- below: a pawn $P_{m}$ is below another pawn $P_{n}$ if the square of $P_{m}$ is $(\mathrm{i}, \mathrm{j})$ and the square of $P_{n}$ is $(\mathrm{k}, \mathrm{j})$ with $k>i$.
- left side: a pawn $P_{m}$ is at left side of another pawn $P_{n}$ if the square of $P_{m}$ is $(\mathrm{i}, \mathrm{j})$ and the square of $P_{n}$ is ( $\mathrm{i}, \mathrm{k}$ ) with $j<k$.
- right side: a pawn $P_{m}$ is at right side of another pawn $P_{n}$ if the square of $P_{m}$ is $(\mathrm{i}, \mathrm{j})$ and the square of $P_{n}$ is $(\mathrm{i}, \mathrm{k})$ with $j>k$.
- above at left/right side: a pawn $P_{m}$ is above at left (right) side of another pawn $P_{n}$ if the square of $P_{m}$ is $(\mathrm{i}, \mathrm{j})$ and the square of $P_{n}$ is $(\mathrm{p}, \mathrm{k})$ with $i>p$ and $j<k$, respectively $k<j$.
- below at left/right side: a pawn $P_{m}$ is below at left (right) side of another pawn $P_{n}$ if the square of $P_{m}$ is $(\mathrm{i}, \mathrm{j})$ and the square of $P_{n}$ is $(\mathrm{p}, \mathrm{k})$ with $i<p$ and $j<k$, respectively $k<j$.
These positions are exemplified in Figure 4.


Fig. 4. Example of pawn position

We may introduce in the knowledge piece some relations that indicate how many empty lines or columns are next to some pawn. These relations are not compulsory. They make the description of the positions of the pawns more accurate. If the knowledge piece contains such relations then the set $S$ of the objects will include a reference to such line(s) or column(s).

The description can be made in a natural language. To define the syntax of the language we have constructed the following $R T N$. As can be seen in Figure 5 , the $R T N$ is composed by eight subnetworks: the subnetwork $S$ defines the syntax of a sentence in our language, the $N p_{-} s g$ and $N p_{-} p l$ are subnetworks for noun phrase, $V p_{-} s g$ and $V p_{-} p l$ are subnetworks for verb phrase, Place realizes a connection between some subnetworks, $A p$ is subnetwork for adverb phrase and the Side subnetwork defines the structure of the side entities.

In what follows we will give an example of an arrangement and a possible knowledge piece that describe it:

The pawn $P_{3}$ is at right side of $P_{1} . P_{1}$ is below $P_{2}$ at left side. $P_{2}$ is below $P_{4}$ at left side. $P_{4}$ is at right side of $P_{5}, P_{5}$ is above $P_{3}$ at left side. An empty line is below $P_{2}$. An empty line is above $P_{4}$.

The labelled graph illustrating this piece of knowledge is $G=\left(S, L_{0}, T_{0}, f_{0}\right)$, where:


Fig. 5. The syntax defined by a RTN

- the set of objects is $S=\left\{P_{1}, P_{2}, P_{3}, P_{4}, P_{5}\right.$, one_line $\}$
- the set of the binary relations is $T_{0}=\left\{\rho_{1}, \rho_{2}, \rho_{3}, \rho_{4}, \rho_{5}\right\}$ and:
$\rho_{1}=\left\{\left(P_{3}, P_{1}\right),\left(P_{4}, P_{5}\right)\right\}, \rho_{2}=\left\{\left(P_{1}, P_{2}\right),\left(P_{2}, P_{4}\right)\right\}, \rho_{3}=\left\{\left(P_{5}, P_{3}\right)\right\}$, $\rho_{4}=\left\{\left(\right.\right.$ one_line,$\left.\left.P_{2}\right)\right\}, \rho_{5}=\left\{\left(\right.\right.$ one_line,$\left.\left.P_{4}\right)\right\}$
- the set of labels $L_{0}=\{R, B, A, B+L, A+L\}$, where:
- the symbol $R$ describes the semantics is at right side of
- the symbol $B$ is for the meaning is below
- the symbol $A$ is for the meaning is above
- the symbol $B+L$ is associated to is below at left side
- the symbol $A+L$ is associated to is above at left side
- the function $f_{0}: L_{0} \longrightarrow T_{0}$ is a bijective mapping because for each label $e$ of $L_{0}$ there is an unique binary relation $r \in T_{0}$ such that $f_{0}(e)=r$ :

$$
f_{0}(R)=\rho_{1}, f_{0}(B)=\rho_{4}, f_{0}(A)=\rho_{5}, f_{0}(B+L)=\rho_{2}, f_{0}(A+L)=\rho_{3}
$$

The labelled graph obtained is represented in Figure 6.


Fig. 6. Labelled graph of $K P$

In order to obtain a labelled stratified graph ([3]) the following computations are realized:

- $T_{0}=\left\{\rho_{1}, \rho_{2}, \rho_{3}, \rho_{4}, \rho_{5}\right\}$, where

$$
\begin{aligned}
& \rho_{1}=\left\{\left(P_{3}, P_{1}\right),\left(P_{4}, P_{5}\right)\right\}, \rho_{2}=\left\{\left(P_{1}, P_{2}\right),\left(P_{2}, P_{4}\right)\right\}, \rho_{3}=\left\{\left(P_{5}, P_{3}\right)\right\}, \\
& \rho_{4}=\left\{\left(\text { one_line }, P_{2}\right)\right\}, \rho_{5}=\left\{\left(\text { one_line }, P_{4}\right)\right\}
\end{aligned}
$$

- $T_{1}=T_{0} \cup\left\{\rho_{6}, \rho_{7}, \rho_{8}, \rho_{9}, \rho_{10}, \rho_{11}\right\}$, where
$\rho_{6}=u\left(\rho_{2}, \rho_{2}\right)=\left\{\left(P_{1}, P_{4}\right)\right\}, \rho_{7}=u\left(\rho_{2}, \rho_{1}\right)=\left\{\left(P_{2}, P_{5}\right)\right\}$,
$\rho_{8}=u\left(\rho_{1}, \rho_{3}\right)=\left\{\left(P_{4}, P_{3}\right)\right\}, \rho_{9}=u\left(\rho_{3}, \rho_{1}\right)=\left\{\left(P_{5}, P_{1}\right)\right\}$,
$\rho_{10}=u\left(\rho_{1}, \rho_{2}\right)=\left\{\left(P_{3}, P_{2}\right)\right\}, \rho_{11}=u\left(\rho_{5}, \rho_{1}\right)=\left\{\left(\right.\right.$ one_line,$\left.\left.P_{5}\right)\right\}$
$\rho_{5}=u\left(\rho_{4}, \rho_{2}\right)$
- $T_{2}=T_{1} \cup\left\{\rho_{12}, \rho_{13}, \rho_{14}, \rho_{15}, \rho_{16}, \rho_{17}\right\}$, where

$$
\begin{aligned}
& \left.\rho_{11}=u\left(\rho_{4}, \rho_{7}\right)\right\}, \rho_{12}=u\left(\rho_{5}, \rho_{8}\right)=u\left(\rho_{11}, \rho_{3}\right)=\left\{\left(\text { one_line }, P_{3}\right)\right\}, \\
& \rho_{13}=u\left(\rho_{11}, \rho_{9}\right)=\left\{\left(\text { one_line }, P_{1}\right)\right\}, \rho_{14}=u\left(\rho_{6}, \rho_{1}\right)=\left\{\left(P_{1}, P_{5}\right)\right\}, \\
& \rho_{15}=u\left(\rho_{8}, \rho_{1}\right)=\left\{\left(P_{4}, P_{1}\right)\right\}, \rho_{16}=u\left(\rho_{10}, \rho_{2}\right)=\left\{\left(P_{3}, P_{4}\right)\right\}, \\
& \rho_{17}=u\left(\rho_{10}, \rho_{7}\right)=\left\{\left(P_{3}, P_{5}\right)\right\}
\end{aligned}
$$

- We append now

$$
\rho_{13}=u\left(\rho_{12}, \rho_{1}\right)=u\left(\rho_{5}, \rho_{14}\right)
$$

Now we can compute the set $L$ and the values of the mapping $f: L \longrightarrow T$, where $T=T_{2}$ :

- $f(R)=f_{0}(R)=\rho_{1} ; f(B)=f_{0}(B)=\rho_{4} ; f(A)=f_{0}(A)=\rho_{5}$;
$f(B+L)=f_{0}(B+L)=\rho_{2} ; f(A+L)=f_{0}(A+L)=\rho_{3} ;$
$L_{1}=L_{0} \cup\{B+L, A+L\}$
- $f(\sigma(B+L, B+L))=\rho_{6} ; f(\sigma(B+L, R))=\rho_{7}$;
$f(\sigma(R, A+L))=\rho_{8} ; f(\sigma(A+L, R))=\rho_{9} ;$
$f(\sigma(R, B+L))=\rho_{10} ; f(\sigma(A, R))=\rho_{11}$;
$f(\sigma(B, B+L))=\rho_{5} L_{2}=L_{1} \cup\{\sigma(B+L, B+L), \sigma(B+L, R)$,
$\sigma(R, A+L), \sigma(A+L, R), \sigma(R, B+L), \sigma(A, R), \sigma(B, B+L)\} ;$
- $f(\sigma(B, \sigma(B+L, R)))=\rho_{11}$;
$f(\sigma(A, \sigma(R, A+L)))=f(\sigma(\sigma(A, R), A+L))=\rho_{12}$;
$f(\sigma(\sigma(A, R), \sigma(A+L, R)))=\rho_{13} ; f(\sigma(\sigma(B+L, B+L), R))=\rho_{14} ;$
$f(\sigma(\sigma(R, A+L), R))=\rho_{15} ; f(\sigma(\sigma(R, B+L), B+L))=\rho_{16}$;
$f(\sigma(\sigma(R, B+L), \sigma(B+L, R)))=\rho_{17}$;
$L_{3}=L_{2} \cup\{\sigma(B, \sigma(B+L, R)), \sigma(A, \sigma(R, A+L)), \sigma(\sigma(A, R), A+L)$,
$\sigma(\sigma(A, R), \sigma(A+L, R)), \sigma(\sigma(B+L, B+L), R), \sigma(\sigma(R, A+L), R)$,
$\sigma(\sigma(R, B+L), B+L), \sigma(\sigma(R, B+L), \sigma(B+L, R))\}$
- $f(\sigma(\sigma(\sigma(A, \sigma(R, A+L))), R))=f(\sigma(\sigma(\sigma(\sigma(A, R), A+L)), R))=$ $f(\sigma(A, \sigma(\sigma(B+L, B+L), R)))=\rho_{13} ;$
$\left.L=L_{3} \cup\{\sigma(\sigma(\sigma(A, \sigma(R, A+L))), R)), \sigma(\sigma(\sigma(\sigma(A, R), A+L)), R)\right)$, $\sigma(A, \sigma(\sigma(B+L, B+L), R))\}$

Now we obtained the labelled stratified graph $K=(G, L, T, u, f)$ over $G=$ $\left(S, L_{0}, T_{0}, f_{0}\right)$.

In what follows we will give a short presentation of the algorithm we used in order to obtain the right positions of the pawns. It is based on the binary relations of the set $T$ (the set $T$ is also named the environment for reasoning). Before to present the algorithm we have to specify the notations we used in:

- for every pawn $P_{i}$ with $1 \leq i \leq 5$ we note by $l_{i}$ and $c_{i}$ the line, respectively the column, of the pawn $P_{i}$.
- $\rho_{p}$ is a below relation if there exists $l \in L$ such that $\rho_{p}=f(l)$ and $l$ is a below label. We define the below labels as follows:
- $B, B+L, B+R$ are below labels
- $\sigma(X, Y)$ is a below label for every below labels $X$ and $Y$
- $\sigma(X, Y)$ is a below label for every below label $X$ and $Y \in\{L, R\}$
- $\rho_{p}$ is a left relation if there exists $l \in L$ such that $\rho_{p}=f(l)$ and $l$ is a left label. We define the left labels as follows:
- $L, A+L, B+L$ are left labels
- $\sigma(X, Y)$ is a left label for every left labels $X$ and $Y$
- $\sigma(X, Y)$ is a left label for left label $X$ and $Y \in\{A, B\}$
- we have $l_{i}<l_{j}$ if there exists a below relation $\rho_{p} \in T$ such that $\left(P_{i}, P_{j}\right) \in \rho_{p}$.
- we have $c_{i}<c_{j}$ if there exists a left relation $\rho_{p} \in T$ such that $\left(P_{i}, P_{j}\right) \in \rho_{p}$.

First of all we determine the lines of the pawns. Then we determine the columns of these pieces taking care that for every pawn $P_{i}$ there must not exist a pawn $P_{j}$ such that $\left|l_{i}-l_{j}\right|=1$ and $\left|c_{i}-c_{j}\right|=1$. Otherwise the pawns could capture between them.

The algorithm to obtain the lines of the pawns is the following:

1. current_line $\leftarrow 1$ and Det_pawns $\leftarrow \emptyset$
2. WHILE $\operatorname{card}($ Det_pawns $)<5$
2.1 for all pawns $P_{i}, l_{i} \leftarrow$ current_line if for every $P_{j} \in \operatorname{Det}$ _pawns we have $l_{j}<l_{i}$
2.2 Det_pawns $\leftarrow$ Det_pawns $\cup\left\{P_{i}\right\}$ and

$$
\text { current_line } \leftarrow \text { current_line }+1
$$

## 3. ENDWHILE

4. If the $K P$ contains empty line(s)-pawn relations, some lines obtained above will be shifted up, if is necessary, in order to accomplish these relations.
where $\operatorname{card}(X)$ denotes the cardinal number of the set $X$.
The algorithm to obtain the columns of the pawns is:
5. current_col $\leftarrow 1$ and Det_pawns $\leftarrow \emptyset$
6. WHILE $\operatorname{card}($ Det_pawns $)<5$
2.1. for all pawns $P_{i}, c_{i} \leftarrow$ current_col if for every $P_{j} \in \operatorname{Det}$ _pawns we have $c_{j}<c_{i}$.
Let $P$ the set of all these pawns $P_{i}$. If for some pawn $P_{i} \in P$ there is a pawn $P_{j} \in$ Det_pawns such that $\left|l_{j}-l_{i}\right|=1$ and $\left|c_{j}-c_{i}\right|=1$ then we delete $P_{i}$ from $P$.
2.2. Det_pawns $\leftarrow$ Det_pawns $\cup P$
2.3. current_col $\leftarrow$ current_col +1 .
7. ENDWHILE
8. If the $K P$ contains empty column(s)-pawn relations, some columns obtained above will be shifted to the right, if is necessary, in order to accomplish these relations.


Fig. 7. The first window of the applet

## 5 Implementation

We have implemented the presented theory in an application that consists of a Java applet and a Prolog file. In this application we used the JIProlog (JavaInternetProlog) product written by Ugo Chirico ([5]) in order to establish a connection between the Java applet and the Prolog file.

The Prolog file uses the RTN presented above in Figure 5 in order to divide each input sentence in the main syntactic categories. Thus, it is easy to obtain the objects and the binary relation between them. These information are the output of the Prolog file for every input sentence that respects the syntax defined in the implemented $R T N$.

The Java applet consists of two windows:

- the first window appears when the applet is loaded. Using the controls of this window the user can:
- enter the Knowledge Piece
- "load" the Knowledge Piece introduced by pressing the Load KP button. As a result, using the JIProlog connection, the Java applet will send all the sentences of the $K P$ to the Prolog file in order to obtain back the relations included.
- see the positions of the pawns determined by the application according with the KP introduced and loaded successfully.
In the figure 7 the solution of the program for the $K P$ presented in the section 4 can be seen. This solution approach very much to the initial image illustrated in Figure 4 with one difference: the column of
the pawn $P_{3}$ is not the right one. This can be explained by the fact that the reasoning environment $T$ does not give very much information about the column of this pawn.
- the second window appears when the DrawGraph button of the first window is pressed. This window contains a group of Color buttons, a canvas in center and another group of buttons: Movement buttons.
The canvas is used for drawing the labelled graph of the relations obtained by the Prolog file. The Color buttons were introduced in the application in order to point out one node and together with it all the arcs that have it as start node. This is done by drawing the chosen node an the arcs with a different color. The other group of buttons can be used for moving the graph inside the canvas.
Because the solution we have implemented for drawing the graph does not guarantee that there is not any crossing arcs, we also gave the possibility of rearranging the graph by dragging the nodes (and together with them all the arcs that start from them are redrawn) and the corners of the broken arcs in the desired position.


Fig. 8. The second window of the applet

## 6 Conclusions and future work

An application of the labelled stratified graphs in image synthesis is presented. Based on a recursive transition network, a given text is analyzed and its
information is extracted. Some labelled stratified graph is constructed and this structure permits us to obtain the image described in the text. If the text does not contain a complete description then a near exact image is obtained. This is illustrated in the example presented in the previous sections. In a future work we shall study the ambiguity of the text and its implications in image synthesis.

## References

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