A Spatial Reasoning HDR System

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Abstract. In this article is presented how HDR systems can be used for spatial reasonings. Because the reasoning entities of the HDR systems are defined based on the inferences they have to perform it is very easy to implement in these systems distributed reasoning processes. For this reason we reconsider our approach presented in [1] for image synthesis reasoning mechanism because this is a distributed reasoning mechanism differentiated along the main axes of a spatial image.
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1 Prerequisites

A **θ-semantic schema** (shortly, θ-schema) is a system $\mathcal{S} = (X, A_0, A, R)$, where:
- X is a finite non-empty set of symbols named **object symbols**
- A_0 is a finite non-empty set of elements named **label symbols** and $A_0 \subseteq A \subseteq \overline{A}_0$, where $\overline{A}_0$ is the Peano θ-algebra generated by A_0
- $R \subseteq X \times A \times X$ is a non-empty set of **relations**.

Let us consider a θ-schema $\mathcal{S} = (X, A_0, A, R)$. We denote by $Ded(\mathcal{S})$ the least set satisfying the following properties:([2])
- If $(x, a, y) \in R_0$ then $([x, y], a) \in Ded(\mathcal{S})$
- If $([x_i, \ldots, x_k], u) \in Ded(\mathcal{S})$ and $([x_k, \ldots, x_r],\ v) \in Ded(\mathcal{S})$, $i < k < r$ and $\theta(u, v) \in A$ then $([x_i, \ldots, x_r],\ \theta(u, v)) \in Ded(\mathcal{S})$. An element of $Ded(\mathcal{S})$ is an **deductive path** of $\mathcal{S}$.

Consider $d_1 = ([x_1, \ldots, x_k], u) \in Ded(\mathcal{S})$ and $d_2 = ([y_1, \ldots, y_m], v) \in Ded(\mathcal{S})$. We write $d_1 \prec d_2$ if $k < m$ and either one of the following condition is satisfied:([2])
- $x_1 = y_1, \ldots, x_k = y_k,\ v = \theta(u, u_1)$
- $x_1 = y_{m-k+1}, \ldots, x_k = y_m,\ v = \theta(v_1, u)$

Definition 1. *([2]) An element $d \in Ded(\mathcal{S})$ is a* **maximal element** *if there is not $\beta \in Ded(\mathcal{S})$ such that: $d \prec \beta$. We denote by $Ded(\mathcal{S})^{max}$ the set of all maximal elements of $Ded(\mathcal{S})$.*

The maximal elements of $Ded(\mathcal{S})$ are named also the maximal paths of the schema $\mathcal{S}$ or the **conclusions** obtained in this reasoning environment.

Let us consider the schemas $\mathcal{S}_1 = (X_1, A_{01}, A_1, R_1)$ and $\mathcal{S}_2 = (X_2, A_{02}, A_2, R_2)$ and $i \in \{1, 2\}$.

If $d_1 = ([x, \ldots, y], u) \in Ded(\mathcal{S}_i)$ and $d_2 = ([y, \ldots, z], v) \in Ded(\mathcal{S}_{3-i})$ then we say that d_1 is **connected to right** by d_2 or d_2 is **connected to left** by d_1. We say that d_1 is **connected** by d_2 if d_1 is connected to right or to left by d_2.

We consider the following sets of deductive paths $L_1 \subseteq Ded(\mathcal{S}_1)$ and $L_2 \subseteq Ded(\mathcal{S}_2)$. We say that $L_1 \cup L_2$ is a **pairwise connected set of deductive paths** if every deductive path of L_i is connected by some deductive path of L_{3-i}.

Using $L_1 \cup L_2$ we can build a new structure over $\mathcal{S}_1$ and $\mathcal{S}_2$, named hyper-schema of first order.

In the resulted hyper-schema each element of L_i, $i \in \{1, 2\}$, is transformed into a **regular arc** by means of a bijective mapping $g_{\mathcal{S}_i}$. More exactly, for $g_{\mathcal{S}_i}(e) = \theta(u, v)$, if $d = ([x, \ldots, y], \theta(u, v)) \in L_i$ then the regular arc (x, e, y) is considered in the hyper-schema.

Definition 2. *([3]) A* ***hyper-schema*** *of order one over $\mathcal{S}_1$ and $\mathcal{S}_2$ obtained by means of L_1 and L_2 is a θ-schema $\mathcal{S}$ that includes the regular arcs obtained from L_1 and L_2.We denote by $Hyp_1(\{\mathcal{S}_1, \mathcal{S}_2\})$ the set of all hyper-schemas of first order over $\mathcal{S}_1$ and $\mathcal{S}_2$.*

In general we write $S \in Hyp_k(\{S_1, S_2\})$ if S_1 and S_2 are hyper-schemas of order $j \leq k-1$ and at least one of them has the order $k-1$.

Definition 3. *([3]) An HDR system is the tuple $H = (Q_1, Q_2, \ldots, Q_k)$ where $k \geq 2$ and*

- *$Q_1 = \{Ag_1, \ldots, Ag_{n_1}\}_{n_1>1}$, constitutes the first level of the system. The entities $\{Ag_1, \ldots, Ag_{n_1}\}$ are named the* ***agents*** *of the system and as structures they are θ-schemas.*
  We note the schemas generated by the agents of the system with $\mathcal{S}_1, \ldots, \mathcal{S}_{n_1}$.
- *$Q_2 = \{KM_{n_1+1}, \ldots, KM_{n_2}\}_{n_2 \geq n_1+1}$, constitutes the set of the knowledge managers of the second level of the system and as structures they are hyper-schemas of order 1.*
  Thus, if we note with $\mathcal{S}_{n_1+1}, \ldots, \mathcal{S}_{n_2}$ the schemas generated by the managers of Q_2 we have that $\forall m \in \{n_1+1, \ldots, n_2\}, \exists m_1, m_2 \in \{1, \ldots, n_1\}, m_1 \neq m_2$ such that:
  $$\mathcal{S}_m \in Hyp_1(\{\mathcal{S}_{m_1}, \mathcal{S}_{m_2}\})$$
- *$Q_j = \{KM_{n_{j-1}+1}, \ldots, KM_{n_j}\}_{j \geq 3}$ represents the set of the knowledge managers for the j-th level of the system. Thus, if we note by $\mathcal{S}_{n_{j-1}+1}, \ldots, \mathcal{S}_{n_j}$ the hyper-schemas generated by the managers of Q_j we have that $\forall m \in \{n_{j-1}+1, \ldots, n_j\}$, $\exists m_1 \in \{n_{j-2}, \ldots, n_{j-1}\}$ and $\exists m_2 \in \{1, \ldots, n_{j-1}\}$, $m_1 \neq m_2$ such that:*
  $$\mathcal{S}_m \in Hyp_{j-1}(\{\mathcal{S}_{m_1}, \mathcal{S}_{m_2}\})$$

2 A spatial reasoning *HDR* system

In order to implement the reasoning mechanism of [1] in a HDR system, the architecture of the system is defined as follows. At the first level there are the agents which start the inputs' processing, each agent being specialized on a single axis' relations. At the upper levels the knowledge managers' tasks consist of enriching the deductions already obtained in the system by combining deductions from different entities, that is, deductions corresponding to different axes of the image. In this manner, *the reasonings entities of this system determine an inference mechanism based entirely on deductions.*

In what follows we consider spatial images with two dimensions. We define an HDR system consisting of two agents at the first level and a single manager at the second level whose task is to combine the agents' deductions in order to illustrate them in a $2D$ picture.

In order to describe arrangements of $2D$ objects in a $2D$ spatial area, each knowledge piece received by the system will contain instances of the following four directional relations: *at left side of, behind, perfectly at left side of, perfectly behind* where the relations *at left side of, perfectly behind* correspond to the Ox axis and *behind, perfectly at left side of* correspond to the Oy axis of the described $2D$ image.

Definition 4. *We define the* **knowledge domain** *of HDR system as the set of the spatial relations existing along the two axes of a 2-dimensional image. It consists of:*

- *the initial relations: (perfectly) at left side, (perfectly) behind*
- *relations that can be derived using composition from the system's initial relations.*

These relations are externally represented by the graphical illustrations of their corresponding spatial semantics.

Definition 5. *We define the architecture of HDR system as follows:*

$$H = (\{Ag_1, Ag_2\}, \{KM_3\})$$

such that $\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$, $i = \overline{1,3}$, $A_{01} = \{L, PB\}$, $A_{02} = \{B, PL\}$. *The labels of* A_{01} *and* A_{02} *correspond to the system's initial relations at* **L***eft side of,* **B***ehind,* **P***erfectly at* **L***eft side of,* **P***erfectly* **B***ehind.*

As we have said, the relations of the system's domain are externally represented by graphical images. More precisely, they are represented by means of some grids that illustrate the spatial relations of some objects.

Definition 6. *Let us consider the HDR system* $H = (\{Ag_1, Ag_2\}, \{KM_3\})$ *with* $\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$, $i = \overline{1,3}$.

We will note by $Y_u(O_1, O_2)$ *the graphical illustration of the spatial relation internally labeled by* u *that exists between the objects* O_1 *and* O_2*, that is:*

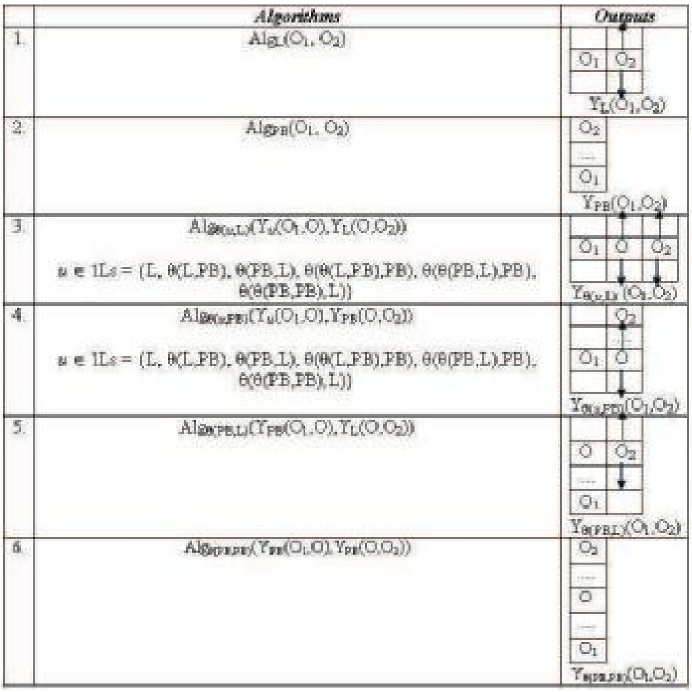


Fig. 1. The external representations corresponding to Ag_1 labels (I).

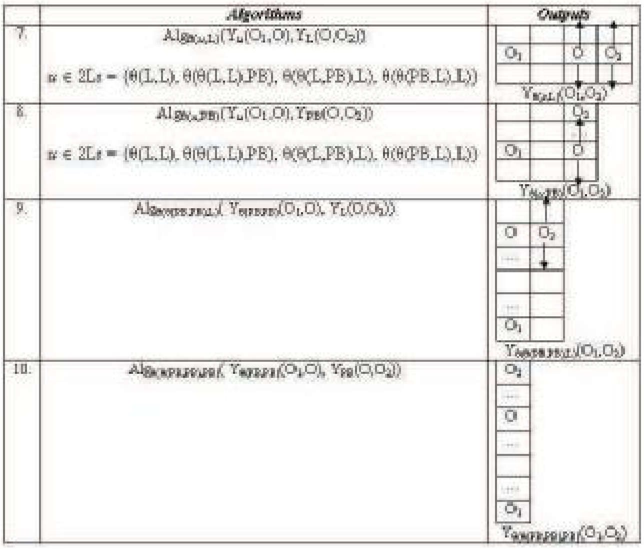


Fig. 2. The external representations corresponding to Ag_1 labels (II).

$$Y_u(O_1, O_2) = Alg_u^{S_i}(O_1, O_2), u \in A_{0i}$$
$$Y_{\theta(u_1,u_2)}(O_1, O_2) = Alg_{\theta(u_1,u_2)}^{S_i}(Y_{u_1}(O_1, O), Y_{u_2}(O, O_2)), u = \theta(u_1, u_2) \in A_i$$

with $i \in \{1, \ldots, 3\}$.

Definition 7. *Let us consider the HDR system* $H = (\{Ag_1, Ag_2\}, \{KM_3\})$ *with* $\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$, $i = \overline{1,3}$.

We say that two labels $u, v \in A_1 \cup A_2 \cup A_3$ *are semantically equivalent if and only if their external representations* $Y_u(O_1, O_2)$ *and* $Y_v(O_1, O_2)$ *are identically,* $\forall O_1, O_2 \in Ob$.

Proposition 1. *Let us consider the HDR system* $H = (\{Ag_1, Ag_2\}, \{KM_3\})$, *with* $\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$, $i = \overline{1,3}$.

If $\exists u, v \in A_k$, $u \neq v$ *such that* $trace(u) = trace(v)$, $k \in \{1, \ldots, 3\}$ *then the labels* u *and* v *are considered semantically equivalent.*

Proof. Let us suppose that $trace(u) = < u_1, \ldots, u_n >_{n \geq 1}$, $trace(v) = < v_1, \ldots, v_m >_{m \geq 1}$. From $trace(u) = trace(v)$ we obtain $n = m$ and $\forall i = \overline{1,n}$: $u_i = v_i$.

If $n \leq 2$ results that $u = v$. For $n = 3$ we will consider $trace(u) = trace(v) = < w_1, w_2, w_3 >$, $w_1, w_2, w_3 \in A_0$. Because $u \neq v$ we can have the following cases:

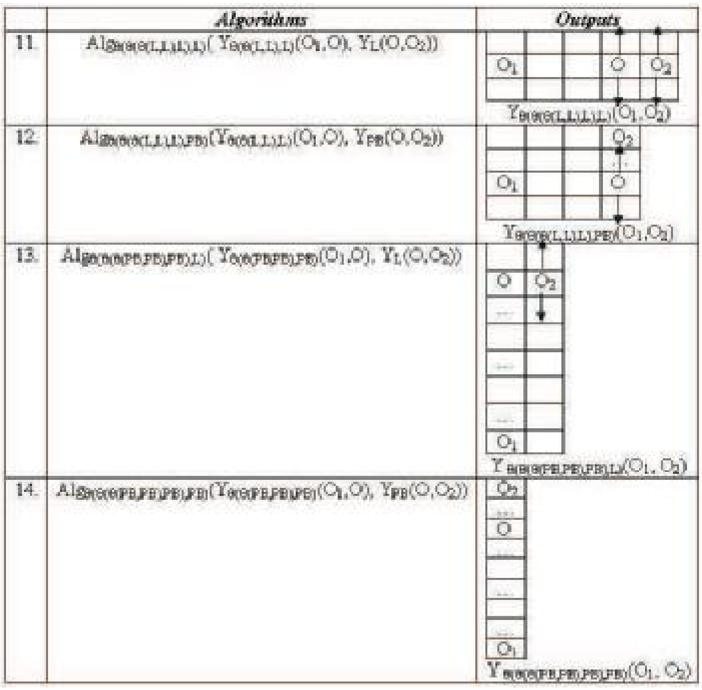


Fig. 3. The external representations corresponding to Ag_1 labels (III).

– $u = \theta(\theta(w_1, w_2), w_3)$ and $v = \theta(w_1, \theta(w_2, w_3))$ or
– $u = \theta(w_1, \theta(w_2, w_3))$ and $v = \theta(\theta(w_1, w_2), w_3)$

Without losing generality, let us suppose that $u = \theta(\theta(w_1, w_2), w_3)$, $v = \theta(w_1, \theta(w_2, w_3))$. Results that both images Y_u and Y_v are made based on the same kind of images Y_{w_1}, Y_{w_2} and Y_{w_3}. Only the order in which these images are combined is different.

Indeed, $\forall O_1, O_2, O_3, O_4 \in Ob$ let us consider $Y_{w_1}(O_1, O_2)$, $Y_{w_2}(O_2, O_3)$, $Y_{w_3}(O_3, O_4)$. Results that $\exists i, j \in \{1, \ldots, 3\}$ such that:
• $Y_{\theta(w_1,w_2)}(O_1, O_3) = Alg^{\mathcal{S}_i}_{\theta(w_1,w_2)}(Y_{w_1}(O_1, O_2), Y_{w_2}(O_2, O_3))$ for $\theta(w_1, w_2) \in A_i$ and thus $Y_{\theta(w_1,w_2)}(O_1, O_3)$ is the graphical illustration of the relations labeled by w_1 and w_2 between O_1 and O_2 and, respectively, between O_2 and O_3
• $Y_{\theta(w_2,w_3)}(O_2, O_4) = Alg^{\mathcal{S}_j}_{\theta(w_2,w_3)}(Y_{w_2}(O_2, O_3), Y_{w_3}(O_3, O_4))$ for $\theta(w_1, w_2) \in A_j$ and thus $Y_{\theta(w_2,w_3)}(O_2, O_4)$ is the graphical illustration of the relations labeled by w_2 and w_3 between O_2 and O_3 and, respectively, between O_3 and O_4

Results that both images $Y_u(O_1, O_4)$ and $Y_v(O_1, O_4)$ illustrates the same relations between the same pairs of objects, which implies their equivalence:

– $Y_{\theta(\theta(w_1,w_2),w_3)}(O_1, O_4) = Alg^{\mathcal{S}_k}_{\theta(\theta(w_1,w_2),w_3)}(Y_{\theta(w_1,w_2)}(O_1, O_3), Y_{w_3}(O_3, O_4))$
– $Y_{\theta(w_1,\theta(w_2,w_3))}(O_1, O_4) = Alg^{\mathcal{S}_k}_{\theta(w_1,\theta(w_2,w_3))}(Y_{w_1}(O_1, O_2), Y_{\theta(w_2,w_3)}(O_2, O_4))$

■

In what follows we consider that the knowledge pieces received by the system are descriptions of $2D$ images consisting of maximum 5 objects. This implies that the labels' lengths of the agents' relations are smaller than 5.

Definition 8. *Let us consider the HDR system* $H = (\{Ag_1, Ag_2\}, \{KM_3\})$, $\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$, $i = \overline{1,3}$, $A_{01} = \{L, PB\}$, $A_{02} = \{B, PL\}$.

The specialization of the system's agents is defined as follows:
• *Ag_1's specialization consists of relations that describe relative positions along the Ox axis, that is, the left side of and perfectly behind relations and all of the relations that can be derived from these ones using composition, that is:*
$\bigcup_{n=0}^{3}(A_1)_n$ *such that:*
$(A_1)_0 = A_{01} = \{L, PB\}$
$(A_1)_n = \bigcup_{u\in(A_1)_{n-1}}\{\theta(u, L), \theta(u, PB)\}_{n\geq 1}$
Results:
$(A_1)_0 = \{L, PB\}$
$(A_1)_1 = \{\theta(L, PB), \theta(PB, L), \theta(PB, PB), \theta(L, L)\}$
$(A_1)_2 = \{\theta(\theta(L, PB), L),\ \theta(\theta(L, PB), PB),\ \theta(\theta(PB, L), L),\ \theta(\theta(PB, L), PB),\ \theta(\theta(PB, PB), PB),\ \theta(\theta(PB, PB), L),\ \theta(\theta(L, L), PB),\ \theta(\theta(L, L), L)\}$
$(A_1)_3 = \{\theta(\theta(\theta(L, PB), L), L),\ \theta(\theta(\theta(L, PB), L), PB),\ \theta(\theta(\theta(L, PB), PB), L),\ \theta(\theta(\theta(L, PB), PB), PB),\ \theta(\theta(\theta(PB, L), PB), L),\ \theta(\theta(\theta(PB, L), PB), PB),\ \theta(\theta(\theta(PB, L), L), L),\ \theta(\theta(\theta(PB, L), L), PB),\ \theta(\theta(\theta(PB, PB), PB), L),\ \theta(\theta(\theta(PB, PB), PB), PB),\ \theta(\theta(\theta(PB, PB), L), L),\ \theta(\theta(\theta(PB, PB), L), PB),\ \theta(\theta(\theta(L, L), PB), L),\ \theta(\theta(\theta(L, L), PB), PB),\ \theta(\theta(\theta(L, L), L), L),\ \theta(\theta(\theta(L, L), L), PB)\}$

The algorithms corresponding to the labels of $\bigcup_{n=0}^{3}(A_1)_n$ *are presented in Figures 1-3.*

• Ag_2*'s specialization consists of relations that describe relative positions along the* Oy *axis, that is, the behind and perfectly left relations and all of the relations that can be derived from these ones using composition:*

$\bigcup_{n=0}^{3}(A_2)_n$ *such that:*

$(A_2)_0 = A_{02} = \{B, PL\}$

$(A_2)_n = \bigcup_{u\in(A_2)_{n-1}}\{\theta(u,B), \theta(u,PL)\}_{n\geq 1}$

Results:

$(A_2)_0 = \{B, PL\}$

$(A_2)_1 = \{\theta(B,PL), \theta(PL,B), \theta(PL,PL), \theta(B,B)\}$

$(A_2)_2 = \{\theta(\theta(B,PL),B),\ \theta(\theta(B,PL),PL),\ \theta(\theta(PL,B),B),\ \theta(\theta(PL,B),PL),\ \theta(\theta(PL,PL),PL),\ \theta(\theta(PL,PL),B),\ \theta(\theta(B,B),PL),\ \theta(\theta(B,B),B)\}$

$(A_2)_3 = \{\theta(\theta(\theta(B,PL),B),B),\ \theta(\theta(\theta(B,PL),B),PL),\ \theta(\theta(\theta(B,PL),PL),B),\ \theta(\theta(\theta(B,PL),PL),PL),\ \theta(\theta(\theta(PL,B),PL),B),\ \theta(\theta(\theta(PL,B),PL),PL),\ \theta(\theta(\theta(PL,B),B),B),\ \theta(\theta(\theta(PL,B),B),PL),\ \theta(\theta(\theta(PL,PL),PL),B),\ \theta(\theta(\theta(PL,PL),PL),PL),\ \theta(\theta(\theta(PL,PL),B),B),\ \theta(\theta(\theta(PL,PL),B),PL),\ \theta(\theta(\theta(B,B),PL),B),\ \theta(\theta(\theta(B,B),PL),PL),\ \theta(\theta(\theta(B,B),B),B),\ \theta(\theta(\theta(B,B),B),PL)\}$

The algorithms corresponding to the labels of $\bigcup_{n=0}^{3}(A_2)_n$ *are presented in Figures 4-5.*

Definition 9. *Let us consider the HDR system* $H = (\{Ag_1, Ag_2\}, \{KM_3\})$, $\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$, $i = \overline{1,3}$, $A_{01} = \{L, PB\}$, $A_{02} = \{B, PL\}$. *The specialization of* KM_3 *is given by the following set:*

$$A_3 = \{\theta(u,v) \mid trace(V(u)) =< u_1,\ldots,u_n >, trace(V(v)) =< v_1,\ldots,v_m >: \\ (u_n = L \wedge v_1 = PL) \vee (u_n = B \wedge v_1 = PB) \vee (u_n \in \{PB, PL\})\}$$

The algorithms corresponding to the relations of KM_3 *are defined as follows:*

- $\forall\theta(u,v)\in A_3$*:* $trace(V(u)) =< u_1,\ldots,u_n >_{n\geq 1}$*,* $trace(V(v)) =< v_1,\ \ldots,\ v_m >_{m\geq 1}$ *with* $u_n = L, v_1 = PL$ *we can have the following two cases for the algorithms* $Alg^{\mathcal{S}_3}_{V(\theta(u,v))}(Y_{V(u)}(O_1,O), Y_{V(v)}(O,O_2))$*:*
  - *if* $V(u) = L$ *then* $Alg^{\mathcal{S}_3}_{V(\theta(u,v))}(Y_L(O_1,O), Y_{V(v)}(O,O_2))$ *appending a new column at the left of* $Y_{V(v)}(O,O_2)$*'s grid such that the object* O_1 *is in* L *relation with the common object* O *of these two grids*
  - *if* $V(u) = \theta(u',L)$ *then* $Alg^{\mathcal{S}_3}_{V(\theta(u,v))}(Alg_{\theta(u',L)}(Y_{u'}(O_1,O'), Y_L(O',O)), Y_V(v)(O,O_2))$ *overdraw the image of* $Y_{\theta(u',L)}(O_1,O)$ *on the image of* $Y_{V(v)}(O,O_2)$ *such that the object* O' *of the first image is in* L *relation with the common object* O
- $\forall\theta(u,v)\in A_3$*:* $trace(u) =< u_1,\ldots,u_n >_{n\geq 1}, trace(v) =< v_1,\ldots,v_m >_{m\geq 1}$ *where* $u_n = B, v_1 = PB$ *we can have the following two definitions for the algorithms* $Alg^{\mathcal{S}_3}_{V(\theta(u,v))}(Y_{V(u)}(O_1,O), Y_{V(v)}(O,O_2))$*:*

Fig. 4. The external representations corresponding to Ag_2 relations (I).

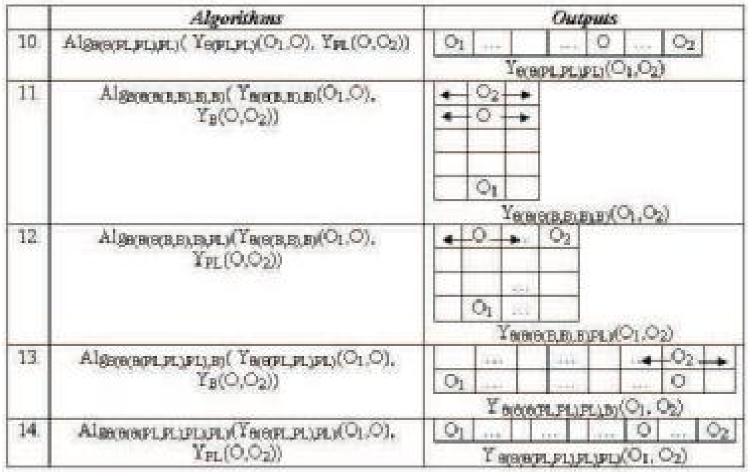


Fig. 5. The external representations corresponding to Ag_2 relations (II).

- *if* $V(u) = B$ *then*
  $Alg^{\mathcal{S}_3}_{V(\theta(u,v))}(Y_B(O_1, O), Y_{V(v)}(O, O_2))$
  appending a new line at the bottom of $Y_{V(v)}(O, O_2)$*'s grid such that the object* O_1 *is in* B *relation with the common object* O *of these two grids*
- *if* $V(u) = \theta(u', B)$ *then*
  $Alg^{\mathcal{S}_3}_{V(\theta(u,v))}(Alg_{\theta(u',B)}(Y_{u'}(O_1, O'), Y_B(O', O)), Y_{V(v)}(O, O_2))$
  overdraw the image of $Y_{\theta(u',B)}(O_1, O)$ *on the image of* $Y_{V(v)}(O, O_2)$ *such that the object* O' *of the first image is in* B *relation with the common object* O

– $\forall \theta(u, v) \in A_3$*:* $trace(u) =< u_1, \ldots, u_n >_{n \geq 1}$ *where* $u_n \in \{PB, PL\}$ *we can define the algorithms* $Alg_{V(\theta(u,v))}{}^{\mathcal{S}_3}(Y_{V(u)}(O_1, O),\ Y_{V(v)}(O, O_2))$ *as follows: for* $V(u) = \theta(u', PB/PL)$ *then*
  $Alg_{V(\theta(u,v))}(Alg_{\theta(u,PB/PL)}(Y_u(O_1, O'), Y_{PB/PL}(O', O)), Y_{V(v)}(O, O_2))$
  overdraw the image of $Y_{\theta(u,PB/PL)}(O_1, O)$ *on the image of* $Y_{V(v)}(O, O_2)$ *such that the object* O' *of the first image is in* PB/PL *relation with the common object* O

2.1 A study case

Let us consider that the inputs of our system describe spatial relations existing between five landmark shapes of a city. A possible inferable knowledge piece KP for the system corresponding to the image from Figure 6 is the following one:

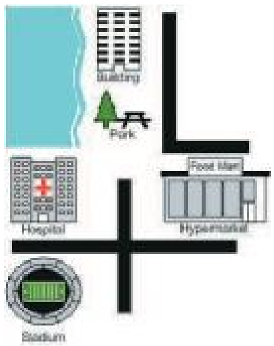


Fig. 6. A $2D$ image of landmark shapes

The Stadium is perfectly behind the Hospital. The Hospital is at left side of the Park. The Park is perfectly behind the Building. The Building is at left side of the Hypermarket.

The Stadium is behind the Hospital. The Hospital is perfectly at left side of the Hypermarket. The Hypermarket is behind the Park. The Park is behind the Building.

Results that the set of KP's objects is { *Hypermarket, Hospital, Stadium, Building, Park* } $\subseteq Ob$.

Fig. 7. The semantic schema corresponding to the relations along the Ox axis

If $ob^{-1}(Stadium) = O_3$, $ob^{-1}(Hospital) = O_2$, $ob^{-1}(Park) = O_5$, $ob^{-1}(Building) = O_4$, $ob^{-1}(Hypermarket) = O_1$, then the internal representations of the knowledge derived from KP are the following ones:

– the agent Ag_1 constructs the semantic schema $\mathcal{S}_1$ where $\mathcal{S}_1 = (X, A_{01}, A_1, R_1)$ (Figure 7) such that:
  $\bullet X = \{O_1, \ldots, O_5\}$
  $\bullet A_{01} = \{L, PB\}$
  $\bullet R_1 = \{$ (O_3, PB, O_2), (O_2, L, O_5), (O_5, PB, O_4), (O_4, L, O_1),

Fig. 8. The semantic schema corresponding to the relations along the Oy axis

$(O_3, \theta(PB, L), O_5)$, $(O_2, \theta(L, PB), O_4)$, $(O_5, \theta(PB, L), O_1)$,
$(O_3, \theta(\theta(PB, L), PB), O_4)$, $(O_2, \theta(\theta(L, PB), L), O_1)$,
$(O_3, \theta(\theta(\theta(PB, L), PB), L), O_1)\}$
• $A_1 = pr_2(R_1)$

– the agent Ag_2 constructs the semantic schema $\mathcal{S}_2$ where $\mathcal{S}_2 = (X, A_{02}, A_2, R_2)$ (Figure 8) such that:
•$X = \{O_1, \ldots, O_5\}$
•$A_{02} = \{B, PL\}$
•$R_2 = \{\ (O_3, B, O_2)$, (O_2, PL, O_1), (O_1, B, O_5), (O_5, B, O_4),
$(O_3, \theta(B, PL), O_1)$, $(O_2, \theta(PL, B), O_5)$, $(O_1, \theta(B, B), O_4)$,
$(O_3, \theta(\theta(B, PL), B), O_5)$, $(O_2, \theta(\theta(PL, B), B), O_4)$,
$(O_3, \theta(\theta(\theta(B, PL), B), B), O_4)\}$
• $A_2 = pr_2(R_2)$

– the knowledge manager KM_3 constructs the semantic schema $\mathcal{S}_3 \in Hyp_1(\{\mathcal{S}_1, \mathcal{S}_2\})$, $\mathcal{S}_3 = (X, A_{03}, A_3, R_3)$ obtained by means of some deductive paths $L_1^{max} \subseteq Ded(\mathcal{S}_1)$, $L_2^{max} \subseteq Ded(\mathcal{S}_2)$ such that $\forall i \in \{1, 2\}$:
L_i^{max} is the set of the maximal deductive paths from $Ded(\mathcal{S}_i)$ that can be connected with the deductive paths of $\mathcal{S}_{3-i}$. Results:
  • $L_1^{max} = \{([O_3, O_2], PB)$, $([O_2, O_5], L)$, $([O_5, O_4, O_1], \theta(PB, L))$,
  $([O_3, O_2, O_5], \theta(PB, L))$, $([O_2, O_5, O_4, O_1], \theta(\theta(L, PB), L))$,
  $([O_3, O_2, O_5, O_4, O_1], \theta(\theta(\theta(PB, L), PB), L))\}$
  • $L_2^{max} = \{([O_3, O_2], B)$, $([O_5, O_4], B)$, $([O_2, O_1, O_5], \theta(PL, B))$,
  $([O_3, O_2, O_1, O_5], \theta(\theta(B, PL), B))$, $([O_1, O_5, O_4], \theta(B, B))$,
  $([O_2, O_1, O_5, O_4], \theta(\theta(PL, B), B))\}$

We will choose the sets L_i, $i = \overline{1, 2}$, as follows:
$L_i = \{d = ([x, \ldots, y], u) \in L_i^{max} \mid \exists d' = ([y, \ldots, z], v) \in L_{3-i}^{max}$,
$\quad trace(u) =< u_1, \ldots, u_n >_{n \geq 1}, trace(v) =< v_1, \ldots, v_m >_{m \geq 1}$:
$\quad (u_n = L \wedge v_1 = PL) \vee (u_n = B \wedge v_1 = PB) \vee (u_n \in \{PB, PL\})\}$

Obviously, we obtain that the set $L_1 \cup L_2$ is a pairwise connected set of deductive paths. For the sets L_1^{max} and L_2^{max} enumerated before we have:
•$L_1 = \{([O_3, O_2], PB)$, $([O_5, O_4, O_1], \theta(PB, L))\}$
•$L_2 = \{([O_2, O_1, O_5], \theta(PL, B))$, $([O_2, O_1, O_5, O_4], \theta(\theta(PL, B), B))\}$.

We take:
•$L_1^a = \{(([O_3, O_2], PB), (O_3, e_1, O_2)), (([O_5, O_4, O_1], \theta(PB, L)), (O_5, e_2, O_1))\}$

$\bullet L_2^a = \{(([O_2, O_1, O_5], \theta(PL, B)), (O_2, m_1, O_5)),$
$(([O_2, O_1, O_5, O_4], \theta(\theta(PL, B), B)), (O_2, m_2, O_4)\}$

Results that the hyper-schema of KM_3 generated by means of L_1 and L_2 is $\mathcal{S}_3 = (X, \{e_1, e_2, m_1, m_2\}, A_3, R_3)$ where:

$R_3 = \{(O_3, e_1, O_2),\ (O_5, e_2, O_1),\ (O_2, m_1, O_5),\ (O_2, m_2, O_4),\ (O_3,\ \theta(e_1, m_2), O_4), (O_2, \theta(m_1, e_2), O_1), (O_3, \theta(e_1, m_1), O_5), (O_3, \theta(\theta(e_1, m_1), e_2), O_1), (O_3, \theta(e_1, \theta(m_1, e_2)), O_1)\}$

$A_3 = pr_2(R_3)$

In what follows we will present the way the outputs of the knowledge manager KM_3 are calculated together with their graphical representations.

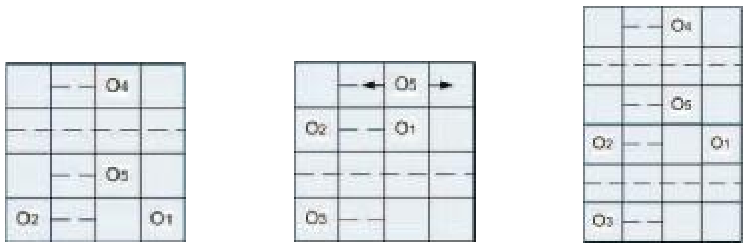
The initial relations of $\mathcal{S}_3$, that is the relations of R_{03} are the following four ones:

- $(O_3, e_1, O_2) \in R_{03},\ e_1 \in pr_2(Arc(L_1))$. Results $\exists d_1 \in Ded(\mathcal{S}_1)$: $T(d_1) = (O_3, e_1, O_2),\ d_1 = ([O_3, O_2], PB)$. Thus $Val_{\mathcal{I}_3}(h([O_3, O_2], e_1)) = Val_{\mathcal{I}_1}(h([O_3,\ O_2],\ PB)) = Alg_{PB}^{\mathcal{S}_1}(O_3, O_2) = Y_{PB}(O_3, O_2)$
- $(O_5, e_2, O_1) \in R_{03},\ e_2 \in pr_2(Arc(L_1))$. Results $\exists d_2 \in Ded(\mathcal{S}_1)$: $T(d_2) = (O_5, e_2, O_1),\ d_2 = ([O_5, O_4, O_1], \theta(PB, L))$. We have $d_2 \Rightarrow_H^* \sigma_1(h([O_5, O_4], PB),\ h([O_4, O_1],\ L))$. Thus $Val_{\mathcal{I}_3}(h([O_5, O_1], e_2)) = Val_{\mathcal{I}_1}(\sigma_1(h([O_5, O_4], PB), h([O_4,\ O_1],\ L))) = Alg_{\theta(PB,L)}^{\mathcal{S}_1}(Alg_{PB}^{\mathcal{S}_1}(O_5, O_4), Alg_L^{\mathcal{S}_1}(O_4, O_1)) = Y_{\theta(PB,L)}\ (O_5, O_1)$
- $(O_2, m_1, O_5) \in R_{03},\ m_1 \in pr_2(Arc(L_2))$. Results $\exists d_3 \in Ded(\mathcal{S}_2)$: $T(d_3) = (O_2, m_1, O_5),\ d_3 = ([O_2, O_1, O_5], \theta(PL, B))$. We have $d_3 \Rightarrow_H^* \sigma_2(h([O_2, O_1], PL),\ h([O_1, O_5], B))$. Thus $Val_{\mathcal{I}_3}(h([O_2, O_5], m_1)) = Val_{\mathcal{I}_2}(\sigma_2(h([O_2,\ O_1], PL), h([O_1,\ O_5],\ B))) = Alg_{\theta(PL,B)}^{\mathcal{S}_2}(Alg_{PL}^{\mathcal{S}_2}(O_2, O_1), Alg_B^{\mathcal{S}_2}(O_1, O_5)) = Y_{\theta(PL,B)}(O_2, O_5)$
- $(O_2, m_2, O_4) \in R_{03},\ m_2 \in pr_2(Arc(L_2))$. Results $\exists d_4 \in Ded(\mathcal{S}_2)$: $T(d_4) = (O_2, m_2, O_4), d_4 = ([O_2, O_1, O_5, O_4], \theta(\theta(PL, B), B))$. We have $d_4 \Rightarrow_H^* \sigma_2(\sigma_2(h\ ([O_2,\ O_1],\ PL),\ h([O_1, O_5], B)),\ h([O_5, O_4], B))$. Thus $Val_{\mathcal{I}_3}(h([O_2, O_4], m_2)) = Val_{\mathcal{I}_2}(\sigma_2(\sigma_2(h([O_2, O_1], PL), h([O_1, O_5], B)), h([O_5, O_4], B))) = Alg_{\theta(\theta(PL,B),B)}^{\mathcal{S}_2}\ (Alg_{\theta(PL,B)}^{\mathcal{S}_2}\ (Alg_{PL}^{\mathcal{S}_2}(O_2, O_1), Alg_B^{\mathcal{S}_2}(O_1, O_5)), Alg_B^{\mathcal{S}_2}(O_5, O_4)) = Y_{\theta(\theta(PL,B),B)}\ (O_2,\ O_4)$

Relations obtained in the hyper-schema $\mathcal{S}_3$ by combining relations of $\mathcal{S}_1$ with relations of $\mathcal{S}_2$:

- $(O_3, \theta(e_1, m_2), O_4) \in R_3$. Results $\exists d_5 \in Ded(\mathcal{S}_3),\ d_5 = ([O_3, O_2, O_4],\ \theta(e_1, m_2))$. We have $d_5 \Rightarrow_H^*\ \sigma_3(h([O_3, O_2], e_1), h([O_2, O_4], m_2))$. Thus $Val_{\mathcal{I}_3}(\sigma_3(h([O_3,\ O_2],\ e_1), h([O_2, O_4], m_2))) = Alg_{V(\theta(e_1,m_2))}^{\mathcal{S}_3}(Val_{\mathcal{I}_3}(h([O_3, O_2], e_1)), Val_{\mathcal{I}_3}\ (h\ ([O_2, O_4], m_2))) = Alg_{\theta(PB,\theta(\theta(PL,B),B))}^{\mathcal{S}_3}(Y_{PB}(O_3, O_2), Y_{\theta(\theta(PL,B),B)}(O_2, O_4)) = Y_{\theta(PB,\theta(\theta(PL,B),B))}(O_3, O_4)$
- $(O_2, \theta(m_1, e_2), O_1) \in R_3$. Results $\exists d_6 \in Ded(\mathcal{S}_3),\ d_6 = ([O_2, O_5, O_1],\ \theta(m_1, e_2))$. We have $d_6 \Rightarrow_H^*\ \sigma_3(h([O_2, O_5], m_1), h([O_5, O_1], e_2))$. Thus $Val_{\mathcal{I}_3}(\sigma_3(h([O_2, O_5], m_1), h([O_5, O_1], e_2))) == Alg_{V(\theta(m_1,e_2))}^{\mathcal{S}_3}(Val_{\mathcal{I}_3}(h([O_2, O_5], m_1)), Val_{\mathcal{I}_3}(h([O_5, O_1], e_2)))= Alg_{\theta(\theta(PL,B),\theta(PB,L))}^{\mathcal{S}_3}(Y_{\theta(PL,B)}(O_2, O_5), Y_{\theta(PB,L)}(O_5, O_1)) = Y_{\theta(\theta(PL,B),\theta(PB,L))}\ (O_2, O_1)$ (see Figure 9(a))

– $(O_3, \theta(e_1, m_1), O_5) \in R_3$. Results $\exists d_7 \in Ded(\mathcal{S}_3)$, $d_7 = ([O_3, O_2, O_5], \theta(e_1, m_1))$. We have $d_7 \Rightarrow_H^* \sigma_3(h([O_3, O_2], e_1), h([O_2, O_5], m_1))$. Thus $Val_{\mathcal{I}_3}(\sigma_3(h([O_3, O_2], e_1), h([O_2, O_5], m_1))) = Alg^{\mathcal{S}_3}_{V(\theta(e_1, m_1))}(Val_{\mathcal{I}_3}(h([O_3, O_2], e_1)), Val_{\mathcal{I}_3}(h([O_2, O_5], m_1))) = Alg^{\mathcal{S}_3}_{\theta(PB, \theta(PL,B))}(Y_{PB}(O_3, O_2), Y_{\theta(PL,B)}(O_2, O_1)) = Y_{\theta(PB, \theta(PL,B))}(O_3, O_5)$ (see Figure 9(b))
– $(O_3, \theta(e_1, \theta(m_1, e_2)), O_1) \in R_3$. Results $\exists d_8 \in Ded(\mathcal{S}_3)$, $d_8 = ([O_3, O_2, O_5, O_1], \theta(e_1, \theta(m_1, e_2)))$. We have $d_8 \Rightarrow_H^* \sigma_3(h([O_3, O_2], e_1), \sigma_3(h([O_2, O_5], m_1), h([O_5, O_1], e_1))$. Thus $Val_{\mathcal{I}_3}(\sigma_3(h([O_3, O_2], e_1), \sigma_3(h([O_2, O_5], m_1), h([O_5, O_1], e_1))) = Alg^{\mathcal{S}_3}_{V(\theta(e_1, \theta(m_1, e_2)))}(Val_{\mathcal{I}_3}(h([O_3, O_2], e_1)), Val_{\mathcal{I}_3}(\sigma_3(h([O_2, O_5], m_1), h([O_5, O_1], e_2))) = Alg^{\mathcal{S}_3}_{\theta(PB, \theta(\theta(PL,B), \theta(PB,L)))}(Y_{PB}(O_3, O_2), Y_{\theta(\theta(PL,B), \theta(PB,L))}(O_2, O_1)) = Y_{\theta(PB, \theta(\theta(PL,B), \theta(PB,L)))}(O_3, O_1)$ (see Figure 9(c))



(a) (b) (c)

Fig. 9. 9(a) shows $Y_{\theta(\theta(PL,B), \theta(PB,L))}(O_2, O_1)$, 9(b) shows $Y_{\theta(PB, \theta(PL,B))}(O_3, O_5)$ and 9(c) shows $Y_{\theta(PB, \theta(\theta(PL,B), \theta(PB,L)))}(O_3, O_1)$

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